MONTHLY NOTICES

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Professor A. FOWLER, F.R.S., PRESIDENT, in the Chair.

James Garton Bower, Erlham House, Norwich ;
Peter Frederick Burns, B.Sc., 64 Queen Bertha Road, Ramsgate;
Miss Marguerite Clare Dunnicliff, Mathematical Tripos, Cambridge, Eversley House, Herne Hill, London, S.E. 24 ;
Willem Jacob Luyten, University Observatory, Leiden, Holland ;
Harold William Newton, Junior Assistant, Royal Observatory, Greenwich, 128 Westcombe Hill, Blackheath, S.E. 3 ; and
Ernest Tourner, The Observatory, Singapore,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Lieut. Stanley Fielder, O.B.E., R.N.R., 64 Great King Street, Edinburgh (proposed by R. A. Sampson) ;
Major E. O. Henrici, R.E., 29 Royal Crescent, London, W. 11 (proposed by Lieut.-Col. H. G. Lyons) ;
William Porthouse, Municipal Officer, 27 Turnbull Road, Longsight, Manchester (proposed by A. L. Cortie) ; and
William Stevens, Royal Observatory, Greenwich, and 17 Roxley Road, Lewisham, S.E. 13 (proposed by Sir F. W. Dyson).

Eighty-one presents were announced as having been received since the last Ordinary Meeting, including, amongst others :—

Lick Observatory Publications, vol. 13 (Studies of Nebulæ), presented by the Observatory ; Publications of the Astronomical Observatory of Michigan, vol. 2, presented by the Observatory ; Tychonis Brahe Opera Omnia, edidit J. L. E. Dreyer, tomus vi., presented by the Editor.

Preliminary Values of Variation of Latitude at Greenwich during 1919. (Plate 4.)

(*Communicated by the Astronomer Royal.*)

The preliminary values of the variation of latitude at Greenwich during 1919, which are here given, are based upon the results of the measurement of 174 plates obtained with the Cookson floating zenith telescope. This number is fewer than the average number of plates obtained per year, the decrease being attributable largely to unfavourable weather conditions in March and December.

During the year the observing programme has been modified, with a view to reducing the mean zenith-distances of each group, which were becoming somewhat large, owing to the effects of precession. Opportunity was taken to remove certain pairs which were found to be either too bright or too faint for accurate measurement. The group connections have consequently become somewhat impaired, and sufficient observations have not yet accumulated for accurate values of the mean zenith-distances of the new pairs to be derived. The definitive results for the latitude-variation for the past year, when obtained, may therefore show greater divergencies from the preliminary values here given than is usual. The changes in the groups may also be responsible for the fact that the mean observed values do not form so smooth a curve as in the last few years.

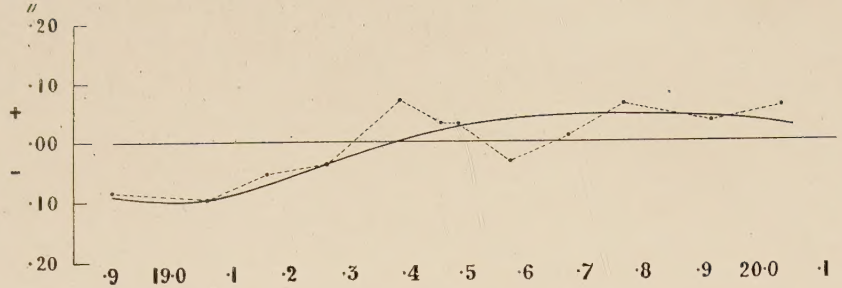
The quantities given in the table are observed *minus* mean latitude.

Latitude Variation at Greenwich.

Date.	Variation.	Number of Plates.
1919·06	−0″·097	10
·16	− ·053	15
·26	− ·039	7
·38	+ ·071	35
·45	+ ·030	17
·48	+ ·028	3
·57	− ·035	14½
·67	+ ·005	24½
·76	+ ·066	25½
·91	+ ·038	15
1920·03	(+ ·058)	5½

The latitude-variation, which was small during the year 1918, decreased still further during last year, and throughout the year it did not exceed ±0″·10. This value is smaller than in any year since 1900. The polar motion being approximately circular, this remark probably applies to both its components, and it would then follow that the actual polar motion during 1919 was the smallest since the year 1900.

From the smooth curve drawn to represent the observed results



Latitude Variation 1919. Greenwich.

Latitude Variation at Matuba 1917–19.

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as closely as possible, the subjoined table of corrections applicable to observed N.P.D.'s is derived.

Corrections applicable to Observed N.P.D.'s.

Date.	Corrn.	Date.	Corrn.
1919 Jan. 1	−0″·10	1919 Aug. 1	+0″·04
Feb. 1	− ·09	Sept. 1	+ ·04
Mar. 1	− ·07	Oct. 1	+ ·04
Apr. 1	− ·04	Nov. 1	+ ·04
May 1	− ·02	Dec. 1	+ ·04
June 1	+ ·01	1920 Jan. 1	(+ ·03)
July 1	+ ·03		

Latitude-Variation Observations at Matuba, Cabinda (Portuguese Congo). By C. Sanders and H. Spencer Jones, M.A., B.Sc. (Plate 4.)

The observations discussed in the present communication were made between 1917 November and 1919 April, when it became necessary to discontinue them on the return of the observer (C. Sanders) to Europe. They were commenced in the hope that, with the instrument available, some work of value might be possible in this direction. As far as the authors are aware, no previous observations for latitude-variation have been carried out so near the equator—the latitude of Cabinda being $-5^\circ 17'$. An extended series of observations in such latitude should give valuable information as to the z-term of the latitude-variation, and it is regretted that it was impossible to continue the present series for a longer period and so strengthen the connections between the several groups. It is hoped, however, that the observations have proved that a comparatively small instrument can be profitably used for latitude-variation work, and that other amateurs, with sufficient enthusiasm, more particularly those living in latitudes in which such observations are not already being carried on at any observatory, may perhaps be induced to make a similar series of observations.

When the observations were commenced, the difficulties to be encountered had not been fully realised. These difficulties arise from the nature of the climate in the Portuguese Congo, which is such that a complete series of observations after the chain-method is almost impossible to obtain; a continuous series is necessary for the accurate determination of the group-corrections and, unfortunately, when the series was commenced this requirement had not been fully realised. Observations had to be limited to the early evening hours, because the atmosphere gets very damp a few hours after sunset, making observations during the later hours of the night almost, or wholly, impossible; then, also, there are the

mosquitoes and other pests. Further, months may at times be lost owing to persistent cloud. The weather conditions during 1918 were unusually favourable, so much so that it is probable that a further attempt to determine latitude-variation at Matuba would probably end in failure. The only suitable climate, in the same latitude, seems to be that of the west coast of South America.

The instrument used was a 35-cm. universal instrument by Sartorius of Göttingen. The "broken" telescope has an aperture of 54 mm., a focal length of 650 mm., and eye-pieces of 11·5 and 9 mm. equivalent focus giving magnifications of about 56 and 72 diameters respectively. The micrometer to which these eye-pieces are adapted has a fixed net of 13 horizontal and 3 vertical (or vice versa) spider threads, of which the centre one is double (distance about 9″), and a movable frame with two spider threads about 11·25 revs. apart, of which one only is used. The adopted value of one revolution of the screw is 63″·0646, which value has been used throughout. It was checked by numerous observations, both on stars and on a collimator.

The observations were made as follows:—Just before and immediately after the observation of the star, the levels were read. The star was pointed once only, at the time of its passage over the middle thread, so that no correction for inclination of the movable thread was necessary, as the collimation of the middle thread never exceeded a few seconds of arc. It would perhaps have been preferable to make more pointings, symmetrically to the middle thread. This, however, was not possible owing to the defective illuminating apparatus available, and to the somewhat close proximity of the side threads to the middle one, their distance being only 20^s. This distance would barely leave sufficient time to read the micrometer and to write the reading down. The disadvantage of only one pointing is, however, compensated to some extent by the greater accuracy of bisection, there being plenty of time to do this without hurry. Care was taken always to move the micrometer screw in the same direction (that of increasing reading). The illumination of the field of view (which is coloured red) was effected by a small hand-lamp, and this had, during the first few months, to serve the double purpose of illuminating the field and of reading levels and micrometer. In 1918 February, a bull's-eye lantern was obtained from Loanda to read the level, etc., enabling the illuminating lamp to be left in position.

*The Observing Programme.**—The stars in the observing programme were selected from Ambronn's *Sternverzeichnis*, the faintest stars included having a magnitude of $6^m·5$, the mean magnitude being about $6^m·0$. A few much brighter stars were included, the brightest having magnitude $3^m·0$. The stars were

* Limitations of space prevent the printing *in extenso* of the details of the observing programme, observations and reductions. Only a descriptive account of these is given; the method of reduction follows closely that usually adopted for such observations.

divided into twelve groups, each group comprising in general 7 pairs of stars, the exceptions being Group IX. with 8 pairs and Group XII. with 9 pairs. The zenith distances in no instance exceeded 24°. The stars in each group were selected so as to satisfy as nearly as possible the following conditions: (1) mean zenith distance of north stars equal to the mean zenith distance of the south stars (within about 2′); (2) the mean right ascensions of the north and south stars equal (within about 4^m); (3) the mean magnitudes of the north and south stars approximately equal (within 1^m). For observing purposes positions were computed from those given in Ambronn's list, which are given to $0^s{\cdot}1$ and 1″. For the reductions, positions were derived for the majority of the stars from the Preliminary General Catalogue of Boss, and for the remainder from the following catalogues: Washington Catalogue of Zodiacal Stars, 1905; Cordoba Second General Catalogue, 1900; Cape Observations, 1905–1908; Abbadia, 1900; Radcliffe, 1900 and 1890; Bordeaux, 1890, and the A.G. Zones (1875, Northern Zones; 1900, Southern Zones). Proper motions, where not obtainable from Boss's Catalogue, were derived from comparison of the modern observations with older catalogues. The proper-motion determinations for some of the southern stars are somewhat weak; for the majority of the groups, the proper motions are only required, however, in order to obtain accurate assumed places, so avoiding not only inconveniently large group-corrections, but also the possibility of periodic errors entering into the assumed declinations and hence into the deduced latitude-variation. Moreover, as the period covered by the observations is comparatively short, it is considered that the proper motions are sufficiently accurate for the purpose.

The twelve groups were distributed as evenly as possible over the twenty-four hours of right ascension, and the observing programme followed very closely that used at the International Latitude Service stations, with the exception that the observations were made soon after sunset instead of round about midnight. The stars of two consecutive groups were observed on one evening, and a change of groups made at approximately monthly intervals.

Errors of Screw.—An accurate determination of the errors of screw, both periodic and progressive, is essential for the satisfactory reduction of the observations. The periodic errors were found to be small and almost insensible, and the mean value of the determinations gave for the corrected reading—

$$u + 0{\cdot}00073 \sin(u - 138°{\cdot}9) + 0{\cdot}00065 \sin(2u - 128°{\cdot}8).$$

The progressive errors were, however, important. They were determined by the method suggested by Boquet in *Les Observations Méridiennes* (Paris, 1909), 2, 235, when he says: "Pour déterminer ϵ_n (irregularité du pas) on pointera la mire avec la fil mobile placé par 0^t en déplaçant la lunette à l'aide de la vis de rappel (petits mouvements), puis on lira le circle. On placera le fil mobile par 1^t et on répétera l'opération. On agira de la même

façon de 0^t à 2^t, de 0^t à 3^t, etc., on obtiendra ainsi les erreurs successives des tours." This was done by pointing an artificial star in the focus of the collimator, but instead of only taking the readings at, say, 0^t and 3^t, the reading of the last position, say n^t, was always added, so that every determination was included between the two extreme positions used, which gave at the same time so many values for the revolution of the screw. In the present case 0^t corresponded to 12^t and n^t to 38^t, the coincidence reading with the middle thread being 25^t. A great number of determinations were necessary in order to obtain the necessary precision, the method being an exceedingly laborious one. A series of determinations were made before the observations were commenced, and it was intended that the errors should be redetermined at their conclusion, using then a Leitz microscope, to which had been adapted a micrometer eye-piece. This would have enabled the displacement of the movable thread of the micrometer to be measured to within 0·0001 inch. It was not found possible, however, to make this proposed redetermination. Between 12^r and 18^r the errors were small and irregular, not exceeding $0^r{\cdot}005$. After 18^r they increased rapidly to $-0^r{\cdot}0263$ at 22^r, after which there was a gradual and almost uniform decrease to 38^r. The results obtained were plotted, the scale of abscissæ being 1 cm. for each revolution and that of the ordinates (representing the correction) of 1 mm. to $0^r{\cdot}001$, so that the correction for each position of the screw could be read with the necessary accuracy. It should be mentioned, however, that as the micrometer readings for each pair were almost identical throughout the series, the progressive error for each pair remains practically unaltered, and any inaccuracies in the determination of the errors therefore go largely into the star-places, since only variations in position are actually required.

The temperature correction of the screw-value was found to amount only to $-0''{\cdot}00097$ for each degree centigrade, and this, in view of the small range of temperature throughout the series, is quite unimportant.

The Levels.—The values of the divisions of the two Talcott levels in terms of one revolution of the screw were found to be—

$$\text{for Level I.} \quad 1 \text{ division} = 0^r{\cdot}01862 \pm 0^r{\cdot}00024,$$
$$\text{,, ,, II.} \quad 1 \text{ ,,} \quad = 0^r{\cdot}01725 \pm 0^r{\cdot}00023.$$

Each level was read before and after the passage of a star through the middle thread, and the mean of the two readings used. The behaviour of Level II. was not entirely satisfactory. When the bubble attained a certain length its indications do not seem to be trustworthy. With changing temperature, therefore, a discordance between the indications of the two levels tends to arise, and it was found that there were always bigger differences between the two levels on the same night when the temperature decreased. The bubble of Level II. also takes a much longer time to come to rest than that of Level I., and it is therefore possible that

in some observations it may not have come entirely to rest when its position was recorded. Late in the series of observations the bubble of Level II. was shortened a little, and its results were then in good agreement with those given by Level I. Although, therefore, the probable errors of the determinations of the division values of the two levels are practically identical, the indications given by Level II. in the latitude observations must be suspected as untrustworthy. It was therefore decided that these should be rejected, and accordingly the results given by Level I., which is good, have alone been used in the reductions.*

The Reductions.—The method of reduction need be only briefly indicated. The assumed declination of each star was reduced to the epoch of the observation, the star corrections being computed in the ordinary way, but the small nutation terms omitted. The mean result, corrected for proper motion, gave the mean declination at the epoch of observation. The mean-level reading was converted into the equivalent in screw-revolutions, added to the screw reading, the correction for the screw errors applied, and the result converted into arc. A correction for refraction, from Albrecht's *Hilfstafeln*, was applied to the value, which, when added to the mean declination previously determined, gave the value of the latitude as determined from the observation of that pair of stars.

The values of the latitude derived from the several pairs of each group were compared, and corrections applicable to the assumed declinations derived so as to reduce each pair to the mean of the group. The mean results provided by the separate groups were then compared, in order to obtain the group-corrections. The determination of these brings in the "closing error." From the totality of the observations, the results in the following table were obtained, I.–II. denoting the difference in latitude deduced from observations of Groups I. and II. on the same nights, and the relative weights being based upon the numbers of pairs used in deriving the means :—

	′	Relative Weight.		″	Relative Weight.
I.–II.	+·120	·071	VII.–VIII.	−·343	·046
II.–III.	−·418	·110	VIII.–IX.	+·246	·058
III.–IV.	+·152	·042	IX.–X.	−·512	·154
IV.–V.	−·185	·024	X.–XI.	+·291	·169
V.–VI.	+·271	·042	XI.–XII.	+·031	·075
VI.–VII.	+·278	·117	XII.–I.	−·200	·092

The closing error, given by the sum of the figures in the second column of the above table, has the value

$$-0''\cdot269.$$

* The accuracy of the results is not much diminished by reading only one level, since the portion of the total probable error of an observation contributed by the levels is very small (*vide* J. W. J. A. Stein, *Ann. der Sternwarte in Leiden*, 9, pt. i. 126, 1906).

In the method of reduction used in the International Latitude Service the closing error is partitioned equally between the twelve differences I.–II. . . . XII.–I., the values being modified so that their sum becomes zero. The cause of the closing error is not known, but it is possibly to be attributed to irregular refraction anomalies. It is therefore plausible to divide it amongst the twelve differences in the proportion of their relative weights; an ordinary least squares solution of the original data, in which the closing error as such is not introduced, demands this partition. It was therefore adopted in the present reductions, and the group corrections, referred to the mean of all groups, were found to be—

I.	−″·03	VII.	+″·32
II.	+·11	VIII.	·00
III.	−·29	IX.	+·26
IV.	−·11	X.	−·23
V.	−·28	XI.	+·09
VI.	+·02	XII.	+·14

Applying these, the final mean latitudes derived from each night's observations were derived and mean values taken for each month during the period covered.

Results. (*a*) *Closing Error.*—The value of the closing error derived above, −0″·269, may be compared with the values obtained at the several stations of the International Latitude Service. The mean value derived from six stations for the period 1906 to 1911 was −0″·224, but there was not only a considerable variation in any given year from one station to another, but also for any given station from one year to another. Only occasionally was a positive closing error obtained, and then the value was small. Negative values exceeding 0″·5 were obtained at Tschardjui in 1906 and 1910, whilst at all the stations except Gaithersburg values exceeding 0″·3 were fairly frequent. At Gaithersburg the closing error was normally less than 0″·2. The magnitude of the closing error obtained at Matuba is, therefore, not exceptional, when compared with the values obtained at the International stations.

In the years 1906–1909, observations for latitude-variation were made at two stations established by the International Latitude Service in the southern hemisphere (latitude −31° 55′), and the mean closing error for the two stations during this period was +0″·147. It was therefore suggested that since the closing error comes out negative along the north parallel and positive in south latitude, there must be a parallel where it would be sensibly equal to zero, and on this hypothesis it was reasonable to anticipate that the equator would be this parallel (*vide* Gratschew,* *Ast. Nach.*, No.

* It should be mentioned, however, that Gratschew assumes that the closing error is entirely due to the assumed value of the aberration-constant (20″·47), not being the correct value. It is now definitely known that this assumption is incorrect.

4742). The present results would appear definitely to disprove this suggestion. An examination of the International results obtained at the two southern stations show, further, that although the closing error was uniformly positive at Oncativo, it was very irregular and on the whole negative at Bayswater. The closing error is intimately bound up with the Kimura-term in the latitude-variation, and is therefore probably dependent in some way upon the local meteorological conditions.

(*b*) *Latitude-Variation.*—The values of the latitude given by the monthly means, after application of the group-corrections, are given in the following table, the weight representing the number of pairs on which the mean is based. Half-weight has been given to pairs observed under bad conditions or for other reasons noted at the time of observation which it was thought might affect their accuracy.

Period.	Mean Date.	Latitude -5° 16′.	Weight
		″	
1917 Oct. 31–Nov. 29	1917·87	59·34	88
Dec. 4–Dec. 30	·95	·35	61
1918 Jan. 9–Jan. 31	1918·06	·50	101
Feb. 4–Feb. 28	·13	·43	84½
Mar. 1–Mar. 26	·20	·47	84
Apr. 1–Apr. 23	·27	·48	69
May 3–May 29	·37	·46	170
June 2–June 30	·46	·58	104
July 1–July 28	·53	·54	72
Aug. 2–Aug. 31	·62	·51	46
Sept. 5–Sept. 30	·72	·40	94
Oct. 3–Oct. 30	·80	·40	53
Nov. 3–Nov. 29	·87	·46	75½
Dec. 3–Dec. 31	·96	·33	58
1919 Jan. 9–Jan. 31	1919·06	·20	68
Feb. 2–Feb. 28	·12	·24	154½
Mar. 1–Mar. 30	·20	·26	124½
Apr. 3–Apr. 20	·27	·30	55

These values, when plotted (see Plate 4), can be represented with fairly good accuracy by a smooth curve. The discordances are somewhat greater than are obtained in the case of the International Latitude Service observations, but against this it must be noted that the number of pairs contributing to each mean are considerably fewer, and that the group-corrections are not nearly so well determined. Considering the size of the instrument and the conditions of observation, the results must be considered

very satisfactory. From the smooth curve definitive values of the latitude can be read: these are given in the following table:—

Definitive Values of Latitude at Matuba.

Date.	Latitude.	Date.	Latitude.
1917·9	−5° 16′ 59″·34	1918·7	−5° 16′ 59″·47
1918·0	·39	·8	·42
·1	·45	·9	·35
·2	·49	1919·0	·28
·3	·52	·1	·24
·4	·54	·2	·26
·5	·54	·3	·31
·6	·52		

The latitude-variation curve is generally similar to that obtained during the corresponding period with the floating zenith telescope at the Royal Observatory, Greenwich. The longitude of Matuba being about 12° E. of Greenwich, the y component of the polar motion does not enter very largely into the latitude-variation, which, in terms of the International x and y, can be represented with sufficient accuracy by $x - 0{\cdot}2y$. The maxima and minima of the latitude curve at Matuba follow those of the Greenwich curve by somewhat over one month.

A lag somewhat less than this might be anticipated from the difference in longitude between the two stations. The most noticeable difference between the two curves is that the maximum near the middle of 1918 is much sharper at Greenwich than at Matuba.

The results do not cover a sufficient period to give much reliable information as to the z-term, and a direct comparison with the International x and y values is not at present possible. Using the Greenwich values of x, however, and adding the small correction arising from the y-term (computed on the supposition that the polar motion can be regarded as approximately circular), a direct comparison with the Matuba results was obtained. After eliminating the Greenwich z-term (as obtained from a comparison of the Greenwich results from 1911 to 1918 with the International values), the residuals, which may be regarded as representing the z-term at Matuba, on being plotted gave a fairly regular simple harmonic curve, with a double amplitude of about 0″·104, with its zero values near the beginning and middle of the year. The mean double amplitude of the z-term at Greenwich is about 0″·071, and at the International stations for the same period about 0″·082. Though the evidence is slender, it appears as though the z-term is larger the lower the latitude. An examination of the Pulkowa observations for latitude-variation from 1905 to 1910 appears to confirm this view, the mean z-term obtained from a comparison with the International x and z values having only 0·54 times the amplitude of the mean z-term for the International stations. The quantities in question are,

however, so small and the variations in them from year to year so considerable that it is doubtful whether too much weight should be attributed to these apparent irregularities. Further observations in low, and particularly in high, latitudes are greatly to be desired.

Accuracy of Results.—The mean-square error of observation of a single pair varies between $\pm 0''.30$ and $\pm 0''.24$, with a general mean of $\pm 0''.26$. This corresponds to a probable error for a single pair of $\pm 0''.17$. The probable error of the latitude deduced from a single night's observations cannot be obtained directly from this figure owing to the "error of the night," so well known in latitude observations.

In order to obtain an estimate of the night error, the latitude-values corresponding to each night's observations were read off from the smooth curve passed through them and the residuals for each group and night obtained. From the arithmetic mean of the whole of these residuals, the probable error of the mean of one complete group was determined as $\pm 0''.080$. The differences of the residuals of the two groups observed on the same evening were then obtained, and from these a probable error of $\pm 0''.072$ was deduced. The difference between these two values is due to the night error, and this has therefore a probable error magnitude of $\pm 0''.035$. This may be compared with values which have been obtained in other series of latitude observations: $\pm 0''.013$ at Gaithersburg (Ross), $\pm 0''.038$ at Leiden (Stein), $\pm 0''.051$ at Greenwich (Jones), and $\pm 0''.068$ at Waikiki (Schlesinger). The evidence supports the view that the magnitude of the night error depends upon the local configuration, and is smaller the more level and symmetrical the land to the north and to the south of the observing station.

From the above determined value of the probable error for a complete group, the probable error of the latitude-value deduced from the mean of a month's observations may be estimated at about $\pm 0''.02$. This is a satisfactorily small value, in view of the smallness of the telescope used and the difficult conditions of observation.

Note on the Sunspot and Facula Disturbances in the Region of the Eclipse Prominence of 1919. By H. W. Newton.

(*Communicated by the Astronomer Royal.*)

The *Astrophysical Journal* for 1919 October contains a paper by Mr. Edison Pettit on "The Great Eruptive Prominences of 1919 May 29 and July 15," in which the author records the connection between these prominences and sunspots as revealed by the spectroheliographic plates taken at the Yerkes Observatory. Mr. Evershed's paper, "The Solar Prominence of 1919 May 29" (*M.N.*, **80**, No. 1), contains additional information derived from

the Kodaikánal disc photographs in Hα light, the study of which suggests that the observation of May 29 indicated not merely an apparent connection, but a definite relationship between the sun-spot and a large system of prominences of which the Eclipse prominence was a member.

Apart from its great dimensions and longevity, this prominence is therefore of exceptional interest, and it appeared desirable to examine in detail the regions of the photosphere adjacent to it for spot and facula disturbances, and especially to follow the life-history of the spot to which the northern extremity of the prominence was connected on May 29. The necessary material for such an investigation would, in the usual way, have been available from the Greenwich Photoheliographic Results, but, owing to large arrears of work, the solar photographs for 1919 have not been measured; moreover, a complete record is not yet available. However, the Greenwich record alone is fairly complete, and preliminary measures have been made on sufficient photographs for the purpose in view.

Spot and Facula Activity in General.

The Eclipse prominence had been observed at the Yerkes Observatory since March 22, when it was seen on the south-west limb of the Sun, extending from latitudes $-35°$ to $-22°$. The photographic record of spots and faculæ revealed no activity at latitude $-35°$, but at the position—latitude $-18°$ and longitude $352°$—there was a regular spot of area 150 millionths, which preceded an area of faculæ. Although the spot had disappeared, this faculous area was still visible during the following rotation, but the region of the southern extremity of the prominence was free from disturbance, and it remained so throughout the entire period during which the prominence was observed. From May 3 to 15 the spot, which was afterwards seen connected with the prominence, was visible for the first time, and May 29 and June 25 brought successive returns of it which are dealt with in detail in the following paragraphs. During the first apparition of this spot group, it was accompanied by a small stream of normal type, distant 6° northwards and really in the same area of disturbance. Apart from this minor group, the nearest spot activity in the northern hemisphere was 80° preceding in longitude; south of the equator, a stable regular spot at $-5°$ preceded the meridian of the prominence spot by 15°, whilst the nearest spot group following was distant 40° in longitude. When the region of the prominence spot returned again to visibility, a stream of normal type was developing nearly on the same meridian at latitude $+22°$, and a small diminishing spot at latitude $+16°$ represented the leader of the small stream seen during the previous rotation. The meridian south of the equator, however, was bare, and a small area only of faint faculæ was measured at latitude $-20°$, longitude 23°, on May 29. For the entire period during which the prominence and the spot were visible, the Greenwich magnetic records of North

Area and Heliographic Position of the Sunspot associated with the Eclipse Prominence of 1919 May.

Date. Greenwich Civil Time, and where taken. G = Greenwich. C = Cape.	Area corrected for Foreshortening (Millionths of Sun's Visible Hemisphere.)		Mean Longitude of Group.		Mean Latitude of Group.	Longitude from Central Meridian.
	Umbra.	Whole Spot.	System I.	System II.		
1919 d			°	°	°	°
May 3·372 C	64	398	5·7	337·6	+6·6	−75·5
4·662 G	82	576	4·7	336·3	+6·6	−58·7
5·464 C	123	698	5·1	336 5	+6·8	−48·4
6·423 G	129	589	5·7	336·8	+6·4	−35·1
7·345 C	120	601	6·2	337·1	+6·6	−22·4
8·508 G	120	595	6·6	337·2	+6·5	− 6·7
9·348 G	104	627	6·7	337·1	+6·5	+ 4·5
10·591 G	106	583	7·0	337·2	+6·6	+21·3
11·355 G	103	562	7·1	337·1	+6·7	+31·5
12·351 G	94	538	7·0	336·8	+6·8	+44·5
13·338 G	84	512	7·0	336·6	+7·1	+57·6
14·337 G	69	457	7·0	336·3	+7·3	+70·8
Means	100	561	6·32	336·88	+6·71	...
May 30·360 G	56	308	9·9	335·9	+6·7	−74·3
31·454 G	63	292	10·1	335·9	+6·8	−59·6
June 1·345 G	57	295	10·2	335·8	+6·9	−47·7
...	...	...	...	...	...	...
3·607 G	47	288	10·8	335·9	+7·3	−17·2
...	...	...	...	...	...	...
5·568 G	41	214	11·0	335·6	+7·3	+ 8·9
6·558 G	42	228	10·8	335·2	+7·4	+21·8
7·361 G	38	208	10·5	334·7	+7·3	+32·2
...	...	...	...	...	...	...
9·500 G	30	206	10·7	334·4	+7·4	+60·7
10·342 G	34	193	10·7	334·2	+7·4	+71·8
Means	45	248	10·52	335·29	+7·17	
June 26·410 G	10	35	14·1	337·6	+8·0	−72·1
...	...	...	...	...	...	...
28·330 G	9	17	14·0	337·1	+8·6	−46·8
29·432 G	9	24	14·2	337·1	+8·8	−32·0
30·556 G	3	10	14·6	337·2	+9·1	−16·7
...	...	...	...	..	...	...
July 2·525 G	8	11	14·8	337·0	+9·7	+ 9·5
3·427 G	2	6	14·9	336·9	+9·7	+21·6
Means	7	17	14·43	337·15	+8·98	...

Note.—Longitude, System I., the daily sidereal motion due to the Sun's rotation is assumed to be 851′·07, irrespective of latitude; this corresponds to Carrington's assumed rotation period of 25·38 days. For System II. the daily sidereal motion is assumed to vary with the latitude in accordance with the formula $866'{\cdot}6 - 128' \sin^2 l$. (Greenwich Photoheliographic Results.)

Force and Declination have been examined. No great disturbance occurred during this period, and of the minor disturbances none could be ascribed in any way to the appearance of the spot group.

Successive Appearances of the Prominence Spot.

First Apparition.—On May 3 the group containing the spot in which the chief interest lies made its appearance on the east limb of the Sun. It had formed when on the invisible hemisphere and was an entirely new disturbance, as no faculæ marked the region when it approached the west limb in the previous rotation. The group was of the stream type, with a large spot as leader and two or three diminishing spots in the rear, the intermediate area containing a large number of very small spots. The train gradually disappeared, and after May 13 the large leader spot alone remained, but a mass of bright faculæ marked the position of the train. The group was most active when first seen, and by May 7 it had become relatively quiescent, and indeed it may be said that throughout it was unmarked by great activity. Between May 3 and 6 the large leader, which appeared near the Sun's limb as a regular spot, combined with an ill-formed spot immediately following it, thus losing its regular shape, but after May 9 it again approximated to the regular form with a complex umbra. The measures of the spot shown in the appended tables indicate its stable condition both in area and position after May 6. The figures, May 3–6, represent the combined measures of the large spot and the smaller component with which it finally coalesced.

Second Apparition. May 29–*June* 11.—No measures could be obtained from the photographs taken on May 29, owing to the proximity of the spot to the Sun's limb. The large spot returned as one of regular type with an occasional small companion. An area of faculæ (about 600 millionths) continued to mark the position of the train. The spot showed no signs of activity during this period; its mean position corresponded closely with that of the first apparition, but its area was diminishing regularly. It may be mentioned that a relative displacement of the umbra towards the centre of the disc, when the spot was near the east and the west limbs of the Sun, clearly indicated that the umbra was below the level of the penumbra.

Third Apparition. June 25–*July* 3.—The spot returned much diminished in size, and it disappeared after July 3; the faculæ accompanying it could be identified during the following rotation. The large proper motion polewards of the spot is somewhat unusual.

Conclusion.

The foregoing description of the life-history of the spot group shows it to have been not an unusual one for the phase of the sunspot cycle, and it is noteworthy that the spot had reached its most stable stage when its actual connection with the prominence was observed. In these respects it was exactly similar to the spot

connected with the prominence of July 15. The region from which the main stem of the Eclipse prominence rose was entirely free from spot and facula disturbances, and the proximity of slight activity near latitude $-20°$ would appear to be only accidental. There is, therefore, but little evidence suggestive of a causal relationship between the spot and the prominence, and nothing to indicate any reciprocal changes between them during their respective developments. In conclusion, I wish to thank Mrs. E. W. Maunder for her suggestions, of which the outcome is this paper.

Royal Observatory, Greenwich:
1920 *Feb.* 20.

On the Observed Changes in the Colour of Jupiter's Equatorial Zone. By A. Stanley Williams.

Normally the colour of *Jupiter's* Equatorial Zone (shortly the E.Z.) is nearly pure white, but towards the end of the year 1869 attention was called by Mr. John Browning, in a note published in the *Monthly Notices*, 30, 39, to a remarkable change in the colour of the zone, which had then assumed a deep ochreish yellow or tawny yellow hue.* This note attracted general attention, resulting in the publication during the next few years of numerous observations of the colour of the zone by many observers. About the same time attempts were made to trace a periodicity in the changes of colour, particularly with one having a period the same as that of the sun-spot cycle. Any such attempt was, however, at that time unavoidably little better than guesswork, on account of insufficient observational material. Now, however, half a century later, we are in a better position, and the object of the present paper is to discuss, with special reference to this question of periodicity, the observed changes in the colour of the E.Z. Since the normal colour of the zone is white, the research is necessarily confined here to the consideration of those exceptional occasions on which the tawny colour has been recorded. Unfortunately, observations of the colour of the zone before the year 1860 are comparatively rare, so that there are many gaps in the record previous to that year. A blank may mean either that the colour was white, and so not noticed particularly, or else simply that there are no observations available at that time. A search through the voluminous mass of observations of *Jupiter* accumulated during the past 150 years led to the result that there were at least five occasions on which the tawny colour had made its appearance in a definite manner. These five occasions appear to be all quite certain and well established. They are considered here in order of date.

* For brevity and convenience the colour has been uniformly termed "tawny" in the present paper, except when quoting observations.

1. 1786 September 30, Schröter describes the colour as a greyish yellowish-brown (*braungelblich graüliche*).* 1790 January 18, the zone appeared to Herschel "of a yellow cast,"† and in 1792 brownish grey.‡ In 1791 Hahn saw the E.Z. as a brownish (*halbbrauner*) streak, and later as full yellow.§ This apparition of the tawny colour must be considered as absolutely certain. It must, indeed, have been a well-developed and striking one to have attracted so much attention at such an early period, and it was perhaps comparable in intensity with the famous apparition of 1869–73. I have assumed the year 1789 for the epoch of maximum, or rather as corresponding nearly to the middle of the observed period of visibility.‖

2. A remarkable series of observations and coloured drawings by Gruithuisen was published in his *Astronomisches Jahrbuch* for 1845. The colours observed chiefly relate to the two dark equatorial belts, but two of the drawings, made in the spring of 1839, indicate that the bright E.Z. was at that time also coloured reddish. In fig. 10 of Plate E the E.Z. is shown with a central red streak traversing it, whilst in fig. 12 the whole zone is drawn filled up with red. The actual colour is described as brown. In the following year (1840) the E.Z. is uncoloured. In 1838 reddish streaks are drawn crossing the E.Z. and joining the two dark equatorial belts. The year 1839 has been adopted for the date of maximum intensity of the tawny colour.

3. 1860 January, Professor A. Herschel made a sketch of *Jupiter*, "noting at the same time the remarkable brown colour of the equatorial belt."¶ About the same time Carpenter, at Greenwich, made a series of forty or fifty drawings of *Jupiter*, and the reddish colour of the central zone was then strong. Apparently Huggins also saw the zone similarly coloured in the years 1858–9 and 1860.** There seems to be some doubt as to when Carpenter's drawings were made. He gives the years as 1861–2, but he may have been speaking from memory only. Ranyard gives the years 1860–1.†† Moreover, Green particularly refers to the long period "from 1860 to 1868" during which the colour was white.‡‡ This supports the inference that the drawings in question were made in 1860–1 rather than in 1861–2. The year 1860 has been adopted as the date of maximum tawny coloration.

4. We now come to the very intense appearance of the tawny

* *Beiträge zu den neusten astronomischen Entdeckungen*, 13.

† *Phil. Trans.*, 1794, 30.

‡ *Astronomical Register*, 9, 136.

§ *Bothkamp Observations*, 2, 93.

‖ For brevity and convenience this and similar dates are in the present paper referred to as the dates of maximum coloration, although in reality they may not have corresponded exactly with the true maximum intensity of the tawny colour. The observations are usually too rough to enable the latter to be ascertained with any certainty.

¶ *M.N.*, 31, 75.

** *Astronomical Register*, 9, 107.

†† *M.N.*, 31. 225.

‡‡ *Mem. R.A.S.*, 49, 270.

coloration in 1869–73. The observations of this period are so numerous that it will be impossible here to consider them separately. Reference may be made to the numerous results published in the *Monthly Notices*, 30, 31, 32, 33, and 34, and the *Astronomical Register*, 8, 9, 10, and 11, but there are also many other observations. Good and very interesting coloured drawings of *Jupiter* by Browning have been reproduced in the *Monthly Notices*, 31, 34 and 32, 321, and in Proctor's *Other Worlds*. Also by Copeland with the Rosse 6-foot reflector in the *Monthly Notices*, 34, 234, and by Trouvelot in Ball's *Story of the Heavens*.

Shortly, it may be stated that the tawny colour seems to have been first noticed in the latter half of the year 1869, and after appearing of extreme intensity during the years 1870, 1871, and 1872, it almost disappeared during 1873. The tawny colour was still, however, noticed from time to time during subsequent years, perhaps down to the year 1878. Apparently there were feeble local or temporary displays of the colour from time to time before it quite disappeared. It is difficult to fix the exact epoch of maximum coloration from the observations. There were probably fluctuations in its intensity from time to time, and in different longitudes. I have assumed 1871 to be the year of maximum intensity.

A very remarkable circumstance was remarked by Browning when comparing a coloured drawing made by himself in 1869 October 24 with two photographs of *Jupiter* taken at De La Rue's observatory almost at the same time.* He noticed that on the negatives the E.Z. was almost absolutely transparent, the light from the tawny zone having failed to act on the sensitive collodion surface. On photographs taken during previous years the same zone had come out almost opaque. Browning suggested that the periodicity of the change in the colour of the zone might be best determined by taking photographs of the planet.

As might be supposed, there are considerable differences in the descriptions of the colour of the zone, seeing that there were many observers, some using refractors, others reflectors, and that the observations were made at different times and under different conditions, and extended over several years. But the differences are not so great as might have been expected. Here are some of the descriptions. "Orange" (Backhouse); "rose tint" (Birmingham); "strong yellow lake," "tawny," and "ochreish" (Browning); "reddish" (Buckingham); "pure deep yellow" and "remarkable yellow tint, like ochreish" (Buffham); "ruddy" and "dusky orange" (Elger); "ruddy" (Gledhill); "coppery tone" (Green); "decided brick-red tint" (Knobel); "brown orange" (Lassell); "pure yellow mixed with crimson" (Mayer); "rich dark brown or tawny" (Salter); "copper colour" (Whitley). These descriptions will be found a useful help in

* *M.N.*, 31, 34.

endeavouring to trace the visibility of the tawny coloration on other occasions.

5. In the spring of 1897 I observed a strong rosy tinge about the southern half of the E.Z. and the S. equatorial belt. The appearance at the time was strongly suggestive of a rose-tinted haze overlying both the belt and the southern half of the E.Z.* The equatorial band, traversing the zone nearly in the middle, was at the same time tinted a very intense, bright, salmon colour. Three photographs (positives) of *Jupiter* taken by Schaeberle nearly at the same time† show the southern half of the E.Z. relatively quite dark, with the equatorial band strongly marked. We have here much the same thing as was pointed out by Browning in 1869, and already alluded to. The rosy tint over the southern half of the E.Z. and the salmon pink of the equatorial band had in great measure failed to act on the sensitive film of the negatives. The rosy tint of the southern half of the E.Z. shortly faded away, but the salmon pink of the equatorial band remained. I do not seem to have recorded the colour of the E.Z. in 1898, but in 1899 I have many notes of its colour, which was generally described as orange or orange yellow, one note stating that "tawny" well expressed the exact tint.

In the 6th, 7th, and 8th Reports of the directors of the *Jupiter* section of the B.A.A. there are many references to the tawny colour of the E.Z. by many observers in the years 1897, 1898, and 1899. In fact, the coloration seems to have been even better observed than it was in the famous period of 1869–73. It is impossible here to do more than refer the reader to these Reports.‡ The tawny colour was also observed by Nijland in 1897, 1898, and 1899.§ He describes the colour as "braunrot" or "dunkelrot," but sometimes as "rotviolett" and even as "violett." Perhaps the latter would refer to the colour of the bright spots in the zone? The colour was also remarked by Lohse in 1897,‖ and by Rheden in that year and in 1898.¶ In 1898 Comas Sola speaks of the intense reddish yellow or yellowish orange coloration of the zone.** There are also observations by various other observers. I have assumed the year 1898 to be the year of maximum intensity.

We now have the dates of five well-determined epochs of visibility of the tawny coloration of the E.Z., namely, 1789, 1839, 1860, 1871, and 1898. A period of about twelve years is at once suggested by these dates. A more exact solution by the method of least squares yielded the following elements of variation :—

Years.

maximum tawny coloration = 1789·25 + 11·95 E.

* See *Mem. B.A.A.*, 6, 126.
† *Pub. A.S.P.*, 9, 173.
‡ *Mem. B.A.A.*, 6, 103 ; 7, 71 and 8, 67.
§ *Recherches astronomiques de l'Observatoire d'Utrecht*, 4, 10–15.
‖ *Pub. Potsdam Obs.*, 21, 58–61.
¶ *Sirius*, 1897, 126 and 1899, 26.
** *Bull. Soc. Ast. de France*, 1898, 516.

The residuals according to these elements are stated in the last column of the following table :—

		Tawny Coloration.		
No.	E.	Observed.	Calculated.	O−C. Years.
...	−2	1767 (?)	1765·30	...
1	0	1789	1789·25	−0·25
2	4	1839	1837·05	+1·95
3	6	1860	1860·95	−0·95
4	7	1871	1872·90	−1·90
...	8	1884	1884·85	(−0·85)
5	9	1898	1896·80	+1·20
...	10	1909	1908·75	(+0·25)

Considering all the circumstances, these residuals are remarkably and gratifyingly small. For, in the first place, from analogy with the sun-spot cycle and the variations of the long-period variable stars, we should hardly expect to find an exact agreement in any case. Then, moreover, we are not dealing with sharply defined epochs of maximum, but with periods of visibility spread over considerable intervals of time. The period of visibility of the tawny coloration averages about three years. In the case of 1871 it was considerably longer. Then the observations are seldom sufficient to fix exactly either the beginning or end of the period of visibility, or the real time of maximum intensity of the tawny coloration. A perfectly justifiable alteration of some of the dates adopted for the epoch of maximum coloration would result in an even closer agreement. For example, take the case of 1871. Browning in 1872 was positive that the tawny colour was then stronger than he had previously seen it,* and this statement, taken in connection with the slow final fading away of the colour, would fully warrant the shifting of the adopted year of maximum intensity from 1871 to 1872. This would result in a large reduction in the residual for this epoch. I have, however, preferred to keep to the dates originally fixed upon, without regard to the question of periodicity at all, rather than to make what might seem an attempt to force a better agreement between observation and theory. In the present investigation no account has been taken of the fraction of the year in which the observations were made, the rough character of the observations not seeming to warrant the so doing, though this would probably result in a slightly better agreement.

According to the foregoing elements, the tawny coloration should have made its appearance in or about the following years: 1801, 1813, 1825, 1849, 1885, and 1909. That it was not recorded in any of the first four years may reasonably be ascribed to the imperfections of the record. There are simply no observations of the colour, so far as I have been able to ascertain. With

* *M.N.*, 32, 321.

regard to the colour about the year 1885, I have found the following observations:—

1883 February 19, Lohse gives a diagram in which a large part of the E.Z. is marked as being "weisslichrosa."* 1884 February 25, Offord remarked that the E.Z., "of a light grey tinged with red, is not continuous. . . . The light bands of the equator are yellowish white." † 1885 March, Smart states that "the equatorial region is usually of a brownish yellow colour, and is divided through most of its length by a tortuous brownish line." ‡ 1885 March (?), Miller remarks that "the equatorial bright zone showed a very complex structure, seeming to consist of a background of dull brownish or reddish grey." § It is clear, therefore, that the tawny coloration duly made its appearance. That it was not more generally noticed is perhaps due to the presence in the southern part of the zone of many very bright spots. Apparently, however, it was but a comparatively feeble display, not to be compared to the famous outburst of 1869–73. I have assumed 1884 to be the year of maximum intensity.

The case of 1909 is somewhat different. The only direct reference to a warm colour that I have been able to find is by Antoniadi in 1910 May 21. ‖ His coloured drawing of this date certainly shows some reddish colour about the E.Z., whilst the "roses-grisâtre" tint of two streaks crossing the zone is expressly alluded to. On the other hand, the reports of the *Jupiter* section of the B.A.A. state, that in 1908 the zone was "devoid of any distinctive colouring"; that in 1909 it was especially bright, being described by the Director of the section "as presenting the appearance of a brilliant band across a dusky disc." But in 1910 the brilliant aspect had disappeared, and the zone appeared at times to three observers to be enveloped in a faint haze. In 1911 the zone was not conspicuously bright, but had a mottled appearance. ¶ My own observations are in agreement with the foregoing. In 1907 the E.Z. was described as white, in 1909 March and April as very white, and once as being "brighter than any of the zones outside it." I have no notes as to the colour or appearance in 1910 or 1911.

It would almost seem from the foregoing as though there had been a partial failure at least in the tawny colour making its appearance at this epoch. But an observation of mine seems to offer a satisfactory explanation of the whiteness and brightness of the E.Z. in 1908 and 1909. Observing in 1909 February 19, I noticed that the north *tropical* zone "was dull and of an ochrey tinge." Again, in 1909 March 8, I noted that the N. tropical zone was shady and dull, though it only showed on that night a faint

* *Pub. Potsdam Obs.*, 21, 19.
† *Trans. Liv. Ast. Soc.*, No. 2, p. 16.
‡ *Journ. Liv. Ast. Soc.*, 3, 134.
§ *Id.*, p. 135.
‖ *Bull. Soc. Ast. de France*, 1911, 168.
¶ See the Reports of the *Jupiter* section, B.A.A., for these years.

ochrey or tawny hue. There are also notes by other observers which show that the N. tropical zone was abnormally dull about this time; but the following description by Scriven Bolton, of the appearance in 1908, is particularly interesting. The N. tropical zone "was faintly shaded and a trifle less white than the S. tropical zone. The southern half, where it fringed the N.E.B., was deep yellow in colour, and at times ruddy. On a few occasions the entire zone presented a decided yellow hue." *

Now these observations seem to show that in 1908 and 1909 the material which produces or causes the tawny colour, normally in the E.Z., had migrated northwards, so as to cover and colour the N. tropical zone, leaving the E.Z. clear. There would seem to be nothing impossible, or indeed unreasonable, in this. Assuming the tawny colour to be due to an effusion of steam into the upper regions of the planet's atmosphere, as was suggested in 1870, or as an alternative of dust, such as gave rise to our wonderful sunset effects after the Krakatoa explosion, then, on some disturbing influence † arising in the southern hemisphere, the steam or dust-laden vapours, floating above the belts in the upper regions of the atmosphere, might be forced more north than usual, so as to overlie the N. tropical zone instead of the E.Z., thus leaving the latter clear and uncoloured. This would account for the whiteness and brightness of the latter zone in 1908 and 1909. In 1910 the tawny colour appears in some measure at least to have returned to the E.Z., as is indicated by the observations of Antoniadi and the members of the *Jupiter* section of the B.A.A. On this supposition I have assumed 1909 as the date for the maximum intensity of the tawny colour. It should be mentioned that it is most unusual for the N. tropical zone to show any tawny or ruddy colour. In fact, I cannot recall a single instance of its having done so previous to 1908. Usually it is one of the whitest and brightest zones on the planet. Also we have, perhaps, another instance of a shift in the latitude favoured by the tawny colour in the spring of 1897, when, as already described, there was a rose-tinted haze apparently overlying both the S.E.B. and the southern half of the E.Z., but not affecting the northern half of the latter zone.

Calculating backwards from 1789, another period of visibility of the tawny colour should have occurred about the year 1765. Now there are four drawings of *Jupiter* made by Messier in 1767 February, which seem very suggestive.‡ These drawings show a single, broad, dark band on or near the equator. If this band were only coloured, these drawings would agree almost exactly with the well-known tinted drawings of Browning. Messier did not, however, actually notice any colour. He describes the band, indeed, as being "blackish," but the instrument that he

* *M.N.*, 70, 234.

† As a possible disturbing cause the presence of the great South Tropical Disturbance might be suggested.

‡ *Phil. Trans.*, 1767, 458.

used was only a 30-inch Gregorian reflector, with a power of 104. The similarity of aspect is, however, so striking that it seems probable that the tawny colour really was present in that year.

According to the elements, the tawny colour should again be at a maximum towards the end of the present year, and according to my observations the E.Z. has now a distinct pale orange or orange-brown colour. The zone is rather dull-looking, duller than either of the two tropical zones, and has sometimes had a mottled or curdled aspect, due to the presence of many faint or minute details.

It will be seen that out of the eleven occasions, beginning with 1789, when the tawny colour should have made its appearance, no less than seven have been well observed,* and all except one by at least several observers. Of the remaining four we have no knowledge, simply because there are apparently no observations.

The adopted period 11·95 years is so close to that in which *Jupiter* makes a revolution round the sun (11·86 years), that it seems likely that the true period of variation is really identical with the latter, though the lapse of another century may be required before this can be definitely proved. It is also nearly the same as the period of 12·08 years which I found in 1899 for the colour changes of the two dark equatorial belts.† Probably there is some connection between the changes in colour of these two belts and that of the E.Z., and the real period of both may ultimately turn out to be the same. The tawny colour makes its appearance in the E.Z. a little before the planet arrives at its aphelion position; not when it is near perihelion, as one would rather have anticipated. The period of 11·95 years is so much longer than that of the sun-spot cycle (11·1 years), that the idea of any connection between the two must be dismissed.

Summary.

1. As the result of a search through the observations of the past 150 years, it has been ascertained that the tawny coloration has made its appearance in the equatorial zone on seven occasions.

2. These indicate variation in a regular and definite period of 11·95 years.

3. This is so near the planet's period of revolution round the sun that the true period is probably identical with it.

4. The tawny coloration makes its appearance a little before the planet arrives at its aphelion position.

5. On the average the tawny coloration persists for about three years.

* The one now under observation will increase the number to eight.

† *M.N.*, 59, 381. The table of observed redness of the belts on pp. 378–79 ends with the year 1899, but it has been extended by Phillips to the year 1915 (see *J.B.A.A.*, 26, 14). An extensive series of observations of the colours of the belts made by the writer has yet to be reduced and discussed.

6. The next maximum coloration is due towards the end of the present year, and the equatorial zone has now assumed the characteristic tawny colour.

St. Mawes:
1920 *February* 16.

Note added 1920 *April* 3.—The tawny coloration of the E.Z., very pale in January last, has been steadily increasing in intensity, and is now very conspicuous. The zone now appears as a broad, brownish-orange-coloured band and has a striking aspect. Apart from this broad, orange-coloured band the planet is at present remarkably free from colour.

With reference to the period of the sun-spot cycle, it is interesting to note that Prof. Turner by harmonic analysis derived a period of 12·0 years for the interval 1860–1894 (*M.N.*, **74**, 88), and now again finds one of 12·0 years for 1889–1919 (*Observatory*, **43**, 134).

Comets with Small Perihelion Distance, and the Resisting Medium.

By A. C. D. Crommelin.

The study of the motion of the comets that have approached the Sun's surface very closely appears to be the most reliable means of obtaining an upper limit to the possible density of the medium surrounding it. The following list gives the elements of the orbits of all the comets that are known to have approached within a solar radius of its surface:—

Comet.	T.	ω.	Ω.	*i*.	Log *q*.	Period.	First Seen.	Last Seen.
		° ′	° ′	° ′		y		
1680	Dec. 17·99	350 40	272 10	60 40	7·7940	8814	1680 Nov. 13	1681 Mar. 20
1843 I.	Feb. 27·41	82 35	1 15	144 19	7·7434	533	1843 Feb. 27	1843 Apr. 19
1880 I.	Jan. 27·62	86 18	6 10	144 40	7·7395	?	1880 Feb. 4	1880 Feb. 19
1882 II.	Sept. 17·23	69 35	346 1	142 0	7·8890	772	1882 Sept. 1	1883 June 1
1887 I.	Jan. 11·34	65 22	339 38	137 37	7·7391	?	1887 Jan. 18	1887 Jan. 29

The element that is liable to most serious doubt is, of course, the period. Professor Newall, however, in his paper (*M.N.*, **80**, 22) has exaggerated the uncertainty in the case of 1843 I. by omitting to note that some of the orbits that he quoted were in no sense definitive, but were merely first approximations derived from short arcs. But the quantity that we desire to obtain at present is not the period, but the amount of possible change of velocity on passing the Sun. Now, the very reason that makes the period difficult to determine, enables us to assert that the loss of velocity in all these cases was very small: except in the third and fifth comets, the period of observation was long enough to enable us

to assert that the velocity of recession fell very little short of the parabolic velocity. We can assert with reasonable confidence that the velocity of approach to the Sun did not exceed the parabolic value. It would suffice for this purpose to refer to the fact that an appreciable excess above that value has not been found among the hundreds of comets whose orbits have been investigated; but we can push the matter a step further by noting the close similarity of the elements of the last four comets on the list, which points unmistakably to a common origin, and that origin within the Sun's domain; for if they had hyperbolic velocities, it would mean that they entered that domain from without, not sharing in the Sun's motion through space, in which case their perihelion distances would have been widely different from each other after such long intervals.

One feature common to all these comets was their great brilliance about the time of perihelion, so that in most cases they were independently discovered by a large number of observers in different countries. This fact makes it unlikely that their return could take place without detection; if we assume this to be the case, then we may say that the period of 1680 is not less than 240 years, and that of 1843 I. not less than 77 years. 1882 II. was well observed for such a long time that we can say with confidence, from the observations alone, that its period is several centuries. 1880 I. and 1887 I. were only observed for a few days, so that I do not make much use of them. But even in their case, the fact that they emerged from the Sun with anything approaching parabolic velocity is eloquent testimony to the infinitesimal character of the resistance that they encountered. We can at least say that their periods are not less than 40 and 33 years respectively.

The distances from the Sun's centre (as seen from Sobral) of the three nearest stars in the eclipse of 1919 May 29 were as follows: Sun's radius 15′·78, 67 Tauri 31′·39, 247 B Tauri 32′·07, κ Tauri 37′·07. The logarithms of the least distances of their rays from the Sun's centre, in astronomical units, are 7·96642, 7·97573, 8·03865 respectively. The length of time during which the comets were inside the sphere with log radius 8·03865 is 2·110 hours for 1680, 2·126 hours for 1843 I., 1·961 hours for 1882 II. Throughout this period the velocity was nearly or quite 300 miles per second. When we reflect that, at a height of 100 miles above the Earth's surface, meteors whose speed is only about 30 miles per second are heated to incandescence in a few seconds, and further, that resistance is considered to vary as the square, if not a higher power, of the velocity, we get an idea of the amount of resistance that would be experienced in two hours with a tenfold greater speed. In fact, we can safely say that the comets would not get through at all if the density of the medium even distantly approached that of our atmosphere at a height of 100 miles. But, in fact, they emerge without any appreciable diminution of velocity.

Taking the formula $\frac{v^2}{\mu} = \frac{2}{r} - \frac{1}{a}$, let r be the radius of the sphere we have been considering, *i.e.* $\log r = 8{\cdot}03865$. Let us in each case assume that the comet enters the sphere with parabolic velocity. If its period on emergence is 772 years (Comet 1882 II.), log $a = 1{\cdot}9251$, and the loss of velocity is 1 in 30800.

If its period is 240 years (assumed minimum value for 1680 Comet), the loss of velocity is 1 in 14100.

If its period is 77 years (assumed minimum value for 1843 I.) the loss of velocity is 1 in 6624.

While for 40 and 33 years (minimum values for 1880 I. and 1887 I.), the losses are 1 in 4280 and 1 in 3764 respectively.

Thus we can assert with moral certainty that in no case was the diminution of velocity as great as 1 in 4000, while in the one case where the period is determined with fair accuracy by the observations, we obtain 1 in 30,000 as the limit.

The 1882 Comet was accurately observed for ten days before perihelion, and the observations made then fit on quite satisfactorily with the orbit deduced from post-perihelion observations (*M.N.*, **80**, 409), thus affording an independent proof of the negligible nature of the resistance.

On some Remarkable Properties of Diurnal Motion.

By N. Liapin.

1. In 1853 Prestel pointed out* an exceedingly simple method of finding latitude. Differentiating the formula

$$\cos z = \sin \phi \sin \delta + \cos \phi \cos \delta \cos t,$$

we have in general

$$\frac{dz}{dt} = \cos \delta \sin q = \cos \phi \sin a$$

(q denoting the parallactic angle).

In the prime vertical $a = \pm 90^\circ$, whence

$$\left(\frac{dz}{dt}\right)_{p.v.} = \pm \cos \phi \quad . \quad . \quad . \quad . \quad (1)$$

When looking for other differential relations of this type, I found that there exist two systems of similar formulæ, which determine both latitude and declination. Thus for stars at elongation $q = \pm 90^\circ$, whence

$$\left(\frac{dz}{dt}\right)_{e} = \pm \cos \delta \quad . \quad . \quad . \quad . \quad (2)$$

* Prestel, "Höchst einfaches Verfahren die geographische Breite zu bestimmen," *A.N.*, **37**, 281. See also W. Chauvenet, *Spherical and Practical Astronomy*, 1868, **1**, 303.

Differentiating the formula

$$\sin z \cos a = -\cos\phi \sin\delta + \sin\phi \cos\delta \cos t,$$

in the case of azimuth we get in general

$$\frac{da}{dt} = \frac{\cos\delta \cos q}{\sin z}.$$

In the prime vertical $\cos q = \sin\phi \sin z \sec\delta$, at elongation $q = \pm 90°$, in horizon $z = 90°$, $\cos q = \sin\phi \sec\delta$, whence *

$$\left(\frac{da}{dt}\right)_{p.v.} = \sin\phi \quad (3); \qquad \left(\frac{da}{dt}\right)_{e} = 0 \quad (4); \qquad \left(\frac{da}{dt}\right)_{h} = +\sin\phi \quad (5)$$

and therefore

$$\left(\frac{da}{dt}\right)_{p.v.} = \left(\frac{da}{dt}\right)_{h}.$$

(The suffix h denotes the horizon.)

On dividing (3) by (1) we have

$$\left(\frac{da}{dz}\right)_{p.v.} = \pm\tan\phi \quad . \quad . \quad . \quad . \quad (6)$$

On the other hand, from the general formula

$$\frac{da}{dz} = \frac{\cot q}{\sin z}$$

we have

$$\left(\frac{da}{dz}\right)_{e} = 0 \quad (7); \qquad \left(\frac{da}{dz}\right)_{h} = \cot q_{h} \quad (8).$$

2. A system of similar relations may be obtained also in the case of parallactic angle. Differentiating the formula

$$\sin z \cos q = \sin\phi \cos\delta - \cos\phi \sin\delta \cos t,$$

we have in general

$$\frac{dq}{dt} = \frac{\cos\phi \cos a}{\sin z}.$$

In the prime vertical $a = \pm 90°$, at elongation $\frac{\cos a}{\sin z} = -\sin\delta \sec\phi$, in horizon $z = 90°$, $\cos a = \sin\delta \sec\phi$, whence

$$\left(\frac{dq}{dt}\right)_{p.v.} = 0 \quad (9); \qquad \left(\frac{dq}{dt}\right)_{e} = -\sin\delta \quad (10); \qquad \left(\frac{dq}{dt}\right)_{h} = -\sin\delta \quad (11)$$

and therefore

$$\left(\frac{dq}{dt}\right)_{e} = \left(\frac{dq}{dt}\right)_{h}.$$

* Formulæ (3) and (5) are given first by W. Struve in his *Breitengradmes sung in den Ostseeprovinzen Russlands*, 1831, 1, 104.

On dividing (10) by (2) we have

$$\left(\frac{dq}{dz}\right)_e = \mp \tan \delta \quad . \quad . \quad . \quad . \quad (12)$$

On the other hand, from the general formula

$$\frac{dq}{dz} = \frac{\cot a}{\sin z}$$

we have

$$\left(\frac{dq}{dz}\right)_{p.v.} = 0 \quad . \quad . \quad (13); \qquad \left(\frac{dq}{dz}\right)_h = \cot a_h \quad . \quad . \quad (14)$$

For the horizon we have, further,

$$\left(\frac{dz}{dt}\right)_h = \cos \delta \sin q_h = \cos \phi \sin a_h \quad . \quad . \quad (15)$$

3. The values of the fifteen differential coefficients thus obtained are summarised for convenience in the following table :— *

$\left(\frac{dz}{dt}\right)_{p.v.} = \pm \cos \phi,$	$\left(\frac{dz}{dt}\right)_e = \pm \cos \delta,$	$\left(\frac{dz}{dt}\right)_h = \frac{\cos \delta \sin q_h}{\cos \phi \sin a_h}$
$\left(\frac{da}{dt}\right)_{p.v.} = \sin \phi,$	$\left(\frac{da}{dt}\right)_e = 0,$	$\left(\frac{da}{dt}\right)_h = + \sin \phi$
$\left(\frac{dq}{dt}\right)_{p.v.} = 0,$	$\left(\frac{dq}{dt}\right)_e = - \sin \delta,$	$\left(\frac{dq}{dt}\right)_h = - \sin \delta$
$\left(\frac{da}{dz}\right)_{p.v.} = \pm \tan \phi$	$\left(\frac{da}{dz}\right)_e = 0,$	$\left(\frac{da}{dz}\right)_h = \cot q_h$
$\left(\frac{dq}{dz}\right)_{p.v.} = 0,$	$\left(\frac{dq}{dz}\right)_e = \mp \tan \delta,$	$\left(\frac{dq}{dz}\right)_h = \cot a_h .$

Regarding the above system of first differential coefficients as a fundamental one, we proceed now to establish the simplest relations which exist between the second differential coefficients.

4. Calculating the general expressions of second differential coefficients of zenith distance, azimuth, and parallactic angle, we obtain:—

$$\frac{d^2z}{dt^2} = \frac{dq}{dt} \cos \delta \cos q$$

$$\frac{d^2a}{dt^2} = - \frac{\cos \delta}{\sin^2 z} \left(\sin z \sin q \frac{dq}{dt} + \cos q \cos z \frac{dz}{dt} \right)$$

$$\frac{d^2q}{dt^2} = - \frac{\cos \phi}{\sin^2 z} \left(\sin z \sin a \frac{da}{dt} + \cos a \cos z \frac{dz}{dt} \right)$$

$$\frac{d^2a}{dz^2} = - \frac{1}{\sin z} \left(\frac{1}{\sin^2 q} \frac{dq}{dz} + \cot q \cot z \right)$$

$$\frac{d^2q}{dz^2} = - \frac{1}{\sin z} \left(\frac{1}{\sin^2 a} \frac{da}{dz} + \cot a \cot z \right).$$

* The upper sign applies to a west star, the lower to an east star.

The following table, which is quite similar to the preceding one, contains fifteen values of these coefficients for the prime vertical, elongation, and horizon:—

$$\left(\frac{d^2z}{dt^2}\right)_{p.v.} = 0, \qquad \left(\frac{d^2z}{dt^2}\right)_{e} = 0, \qquad \left(\frac{d^2z}{dt^2}\right)_{h} = -\sin\phi\sin\delta$$

$$\left(\frac{d^2a}{dt^2}\right)_{p.v.} = \mp\frac{\cos\phi\sin\delta}{\sin z_{p.v.}}, \quad \left(\frac{d^2a}{dt^2}\right)_{e} = \pm\frac{\sin\delta\cos\delta}{\sin z_e}, \quad \left(\frac{d^2a}{dt^2}\right)_{h} = \sin\delta\cos\delta\sin q_h$$

$$\left(\frac{d^2q}{dt^2}\right)_{p.v.} = \mp\frac{\cos\phi\sin\phi}{\sin z_{p.v.}}, \quad \left(\frac{d^2q}{dt^2}\right)_{e} = \pm\frac{\cos\delta\sin\phi}{\sin z_e}, \quad \left(\frac{d^2q}{dt^2}\right)_{h} = -\sin\phi\cos\phi\sin a_h$$

$$\left(\frac{d^2a}{dz^2}\right)_{p.v.} = \mp\frac{\tan\phi}{\tan z_{p.v.}}, \quad \left(\frac{d^2a}{dz^2}\right)_{e} = \pm\frac{\tan\delta}{\sin z_e}, \quad \left(\frac{d^2a}{dz^2}\right)_{h} = -\frac{\cot a_h}{\sin^2 q_h}$$

$$\left(\frac{d^2q}{dz^2}\right)_{p.v.} = \mp\frac{\tan\phi}{\sin z_{p.v.}}, \quad \left(\frac{d^2q}{dz^2}\right)_{e} = \pm\frac{\tan\delta}{\tan z_e}, \quad \left(\frac{d^2q}{dz^2}\right)_{h} = -\frac{\cot q_h}{\sin^2 a_h}.$$

The following simple relations may be easily derived:—

$$\left(\frac{d^2q/dt^2}{d^2q/dz^2}\right)_{p.v.} = \cos^2\phi, \quad \left(\frac{d^2a/dt^2}{d^2q/dt^2}\right)_{p.v.} = \cos z_{p.v.}, \quad \left(\frac{d^2q/dt^2}{d^2z/dt^2}\right)_{h} = -\tan a_h$$

$$\left(\frac{d^2a/dt^2}{d^2a/dz^2}\right)_{e} = \cos^2\delta, \quad \left(\frac{d^2q/dt^2}{d^2a/dt^2}\right)_{e} = \cos z_e, \quad \left(\frac{d^2a/dt^2}{d^2z/dt^2}\right)_{h} = -\tan q_h.$$

The following simple relations between the first and second differential coefficients in the prime vertical and at elongation

$$\left(\frac{da/dz}{d^2q/dz^2}\right)_{p.v.} = -\sin z_{p.v.}, \qquad \left(\frac{dq/dz}{d^2a/dz^2}\right)_{e} = -\sin z_e$$

$$\left(\frac{da/dz}{d^2a/dz^2}\right)_{p.v.} = -\tan z_{p.v.}, \qquad \left(\frac{dq/dz}{d^2q/dz^2}\right)_{e} = -\tan z_e$$

give

$$\left\{\frac{(d^2q/dz^2)(d^2a/dt^2)}{d^2q/dt^2}\right\}_{p.v.} = \left(\frac{d^2a}{dz^2}\right)_{p.v.} \qquad \left\{\frac{(d^2a/dz^2)(d^2q/dt^2)}{d^2a/dt^2}\right\}_{e} = \left(\frac{d^2q}{dz^2}\right)_{e}.$$

On the horizon:

$$\left(\frac{d^2z/dt^2}{dq/dt}\right)_{h} = \sin\phi, \quad \left(\frac{d^2q/dt^2}{dz/dt}\right)_{h} = -\sin\phi, \quad \left(\frac{da/dz}{d^2q/dz^2}\right)_{h} = -\sin^2 a_h$$

$$\left(\frac{d^2a/dt^2}{dz/dt}\right)_{h} = \sin\delta, \quad \left(\frac{d^2z/dt^2}{da/dt}\right)_{h} = -\sin\delta, \quad \left(\frac{dq/dz}{d^2a/dz^2}\right)_{h} = -\sin^2 q_h.$$

From all these formulæ there clearly stands out the remarkable parallelism between four elements—latitude, declination, azimuth, and parallactic angle. In fact, for stars culminating south of the zenith, latitude and azimuth analytically play the same part as declination and parallactic angle for stars culminating between the pole and the zenith. Azimuth of stars of the first group varies from 0° to 360° as reckoned from upper culmination in the

direction of diurnal motion, whilst parallactic angle for these stars varies from 0° to 0°, having a maximum and a minimum. Parallactic angle of stars of the second group varies from 0° to 360° as reckoned from lower culmination in the direction inverse to that of diurnal motion, whilst azimuth of these stars varies from 180° to 180°, having a minimum and a maximum. Hence arise different signs of all first and second differential coefficients in the prime vertical and elongation, as may be seen from our two tables. Parallactic angle of stars of the first group becomes a maximum or a minimum when their azimuth is 90° or 270° respectively, whereas azimuth of stars of the second group becomes a maximum or a minimum when its parallactic angle is 270° or 90° respectively. Moreover, it is a matter of interest that first differential coefficients of azimuth in the prime vertical define latitude only, those of parallactic angle at elongation declination only. First differential coefficients of azimuth at elongation and of parallactic angle in the prime vertical vanish. In the horizon first differential coefficients of azimuth define latitude and parallactic angle, whilst those of parallactic angle define declination and azimuth.

5. From the inspection of our formulæ two important properties of latitude and azimuth on the one hand, declination and parallactic angle on the other, become obvious, viz. *reciprocity and conjugation.* Thus, our analysis gives a full account of all the circumstances of diurnal motion, thus drawing an interesting law of symmetry between these elements, which at first appearance seem to have no internal connection with each other.

University Observatory,
Odessa, Russia:
1919 *December* 26.

On the Suggested Increase in Period of Variable Stars in Phillips's Group I., with Particular Notes on U Virginis, S Cephei, T Cassiopeiæ, S Ursæ Majoris, R Serpentis, T Hydræ, T Herculis, and other Stars. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In a former paper the exceptional stars of Group II. were shown to have decreasing periods: in the present a corresponding increase of period for some Group I. stars is indicated. The word "exceptional" may be emphasised. The examination of all the stars for which we have light curves good enough to assign the group is not yet completed: but it has gone far enough to show that for the stars in general we have not as yet a long enough series of observations to be sure of the changes in period, which are usually small. We can only note that for the stars which show large changes the evidence seems to point all one way.

2. For the large changes it is natural to go first to Chandler's secular terms, of which those assigned to Group II. were dealt with in the last paper. We now have left the stars in Table I.

TABLE I.

No. Chandler.	No. Harvard.	Name.	Group.	Chandler Formula.
2213	060822	η Geminorum		Period increasing?
4596	124606	U Virginis	I.	$206{\cdot}92E - {\cdot}006E^2$
5856	*161607*	W Ophiuchi		Period decreasing?
6903	*191017*	T Sagittarii		$375{\cdot}8E + 0{\cdot}50E^2$
7779	213678	S Cephei	I.	$485{\cdot}8E + 0{\cdot}05E^2$

It will be seen that there are only two stars in this list definitely assigned to Group I. as yet, and one of them has a negative coefficient for E^2, *i.e.* a decreasing period assigned by Chandler. It will be well to examine these two stars first.

U Virginis 124606. *Adopted Period* 206·92 *Days.*

There are apparently breaks at intervals of 14 cycles. For some of the groups the material is very scanty. The first two residuals at −18 and −16 (values +45 and −4) are omitted as inconclusive.

From −15. *Adopted Period* 207 + 0·7 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
−13	−14	−14	0	− 2	− 6	− 6	0

From −1. *Adopted Period* 207 − 1·0 *Days.*

+ 1	+ 8	+ 9	− 1	+ 7	+ 4	+ 3	+ 1
+ 3	+17	+ 7	+10	+10	− 1	0	− 1
+ 5	− 3	+ 5	− 8	+12	− 2	− 2	0

From +13. *Adopted Correction* +1·5 *Days.*

+14	− 1	−4·8	+ 3	+26	+29	+13·2	+16
+23	−10	+8·7	−19				

From +27. *Adopted Correction* −0·2 *Days.*

+28	+ 4	−3·4	+ 7	+47	+ 7	−7·2	+14
+29	− 3	−3·6	+ 1	+49	− 7	−7·6	+ 1
+30	−12	−3·8	− 8	+51	−22	−8·0	−14

From +55. *Adopted Correction* +2·0 *Days.*

+56	−27	−25	− 2	+65	− 5	− 7	+ 2
+58	−25	−21	− 4	+66	+13	− 5	+18
+60	−12	−17	+ 5	+67	−10	− 3	− 7
+61	−20	−15	− 5	+68	− 2	− 1	− 1
+63	−14	−11	− 3				

From +69. *Adopted Period* 206·92 + 1·0 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
+70	−18	−17	− 1	+75	−13	−12	−1
+72	−12	−15	+ 3	+76	−10	−11	+1
+74	−26	−13	−13				

[The residuals for E+75 and +76 are from *H.A.*, 63, 61.]

Collecting results we get—

	d	d	Changes of Phase.
−15 to − 1	+0·7		
− 1 to +13		−1·0	− 5 to +13 = +18
+13 to +27	+1·5		− 3 to − 6 = − 3
+27 to +55		−0·2	+17 to − 3 = −20
+55 to +69	+2·0		− 9 to −27 = −18
+69 to +83		+1·0	+ 1 to −18 = −19

The periods show an increase with oscillation, and the changes of phase at the breaks are approximately equal, with one exception, which ought probably to be zero. But the material is so scanty that the solution cannot be regarded as definitive: it is rather illustrative, showing that the material is consistent with the interpretation of an increase in period. And attention may further be drawn to the reason of Chandler's term −·006E², which was probably suggested by the negative residuals from E = +51 onwards. On the present hypothesis these are due to a sudden change of phase at E = +55, accompanied by an *increase* of period.

S Cephei 213678. *Adopted Period* 485·8 *Days.*

This is a difficult star to observe, and the accidental errors are therefore large; but on almost any hypothesis the period must have increased.

Omitting the first maximum recorded in *H.A.*, 55, for E = 11, residual +67, we have—

From E = 12. *Adopted Correction to Period* −4 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
14	+31	+29	+ 2	20	−19	+ 5	−24
16	+30	+21	+ 9	21	+ 3	+ 1	+ 2
17	− 2	+17	−19	22	+25	− 3	+28
18	−21	+13	−34	23	+ 9	− 7	+16
19	+27	+ 9	+18	24	−27	−11	−16

From E = 24·5. *Adopted Correction to Period* +7 *Days.*

25	+31	+27	+ 4	29	+49	+55	− 6
26	+48	+34	+14	30	+60	+62	− 2
27	+29	+41	−12	31	+86	+69	+17
28	+40	+48	− 8				

The change takes place abruptly between E = +24 and +25. There is a jump of about 40 days in phase, besides the change in period.

Turning now to the other stars in Table I.

T Sagittarii 191017. Adopted Period 381 *Days.*

Pogson observed maxima at E = −5, −4, −3, and Schönfeld followed with −2, +1, and +2; all of which fit the adopted period of 381 days well: it could not be made longer than 384 days. After this no maxima are recorded in *H.A.*, 55, till E = 25, when the deviation is −146 days and continues large. An adopted period of 396 days would give results as follows:—

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
25	13001	12996	+ 5	28	14175	14184	−9
26	13384	13392	− 8	29	14582	14580	+2
27	13799	13788	+11	38	18145	18144	+1

It is pretty clear that the period has increased by some 15 days. At the moment of writing the group of this star has not been assigned: it will be interesting to know whether it is a Group I. star.

η Geminorum 060822. *Adopted Period* 231·4 *Days.*

This star has apparently not attracted much attention in England as yet, probably because of its small variation, 3·7 to 4·2. But it seems possible that it would well repay observation, and as it is always easily visible to the naked eye, it is an object on which anyone could work. The *minima* tabulated in *H.A.*, 55, show that the variation was quite regular up to E = 21, and that Chandler's period of 231·4 days, adopted for use, was pretty near the truth. But he included the observation at E = −34, which, in view of the later behaviour of the star, possibly belongs to a separate run; and if we omit it, the period should be diminished by about 1·0 day, as below:—

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
−34	+ 4	+37	−33	11	−12	− 8	− 4
0	− 3	+ 3	− 6	12	−18	− 9	− 9
2	+14	+ 1	+13	16	−18	−13	− 5
5	+14	− 2	+16	18	−24	−15	− 9
7	− 2	− 4	+ 2	19	− 2	−16	+14
8	−20	− 5	−15	21	− 9	−18	+ 9
10	−15	− 7	− 8	24	+28	−21	+49

The observation at E = 24 clearly belongs to a new run, which extends to E = 52:—

24	+28	+28	0	28	(+117)	+32	(+85)
26	(−60)	+30	(−90)	51	+53	+55	− 2
27	+46	+31	+15	52	+56	+56	0

The correction to period is adopted as +1·0, and suits four of the observations; but two are conspicuously anomalous. Has this star, as suggested by the experience of W Cygni, a double period?

The observations of maxima do not add much to the above information.

The group has not yet been assigned.

W Ophiuchi 161607.

Only two maxima, at E = 21 and 29, are recorded in *H.A.*, 55. Chandler's note about the decreasing period seems to be based on his own (unpublished?) observations, mentioned in the notes of his third Catalogue.

This paper and the preceding one aim rather at a rapid review of the possibilities, with the object of directing further inquiry, than at a complete scrutiny. It is natural to consider first the stars which had been noted in the last comprehensive survey (which I take to be that of Chandler) as changing their periods, decidedly or problematically; and this has now been done, with the result that no definite evidence against the main hypothesis (stars of Group II. decreasing, of Group I. increasing, their periods) has been encountered, and a considerable amount of positive evidence in favour of it has been found, though several cases have been left in doubt. The next step was to take the stars tabulated by Phillips in the two groups and examine their records one by one. Since the positive evidence obtained from the Chandler scrutiny was much scantier in the case of Group I., it was natural to examine this group in detail first. The following star seems to give clear evidence:—

T Cassiopeiæ 001755. *Adopted Period* 445 *Days.*

The period was originally definitely shorter, say 425 days, implying a correction of −20 days. If we try to take in E = 10, as below, with a correction of −10 days only,

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
0	+55	+48	+7	3	+ 2	+18	−16
1	+35	+38	−3	10	−29	−52	+23
2	+24	+28	−4				

we suit neither the first four, which show a steady decrease, nor the fifth, and the sixth will be much worse. There must be a "break" between E = 3 and E = 10, and then we can suit the first four residuals with a period of 425 days, as stated. The period then changes to about 442 days: it seems possible that there is another break about E = 21, but the evidence is scarcely sufficient to establish it.

From E = 10. Adopted Correction to Period − 3 Days.

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
10	−29	+20	−49	21	− 3	−13	+10
11	+23	+17	+ 6	22	−34	−16	−18
12	−24	+14	−38	23	0	−19	+19
15	+ 3	+ 5	− 2	24	−27	−22	− 5
16	+ 4	+ 2	+ 2	25	−34	−25	− 9
17	+ 9	− 1	+10	26	−32	−28	− 4
18	−11	− 4	− 7	27	−39	−31	− 8
19	+ 3	− 7	+10	28	−41	−34	− 7
20	−13	−10	− 3				

The two large residuals at E = 10 and E = 12 are rather disturbing; but E = 12 is a Baxendell observation, quoted from the *Observatory*, **9**, 159, as in 1885 October 16. On reference to his note-books I find that his observations were too fragmentary to identify the maximum closely—(there are, for instance, none at all in October itself)—and are quite consistent with a max. falling a month later. The E = 10 max. is by Wilsing of Potsdam, and cannot be checked at present.

There seems no reason to doubt the considerable change of period from 425 days to 442 days.

S Ursæ Majoris 123961. *Adopted Period* 226·5 *Days.*

The discussion of this Group I. star has proved particularly interesting for several reasons. It already presented various problems when Pogson's observations were being edited (*Mem. R.A.S.*, **58**, Introd.); it seemed to be an exception to the suggested rule of "increase of period for Group I. stars" when this rule was recently suggested as a speculative outcome of the W Cygni discussion (*M.N.*, *R.A S.*, **80**, 41); and finally, it proved to be puzzling (at first, though fairly simple ultimately) in discussing the long Harvard series of maxima tabulated in *H.A.*, **55**, 166. The ultimate solution will be sufficient here without following the trial steps. As far as E = 14 the period is 223·9 days; but the even run of this period is interrupted by a "jump" of 17 days in epoch after E = − 1. The jump at this point naturally raises the question whether the period was quite the same before and after, and a separate discussion was made for the two portions, with the result that the corrections obtained (to the adopted Harvard 226·5 days) were − 2·66 and − 2·60 days respectively. It was therefore assumed that the period remained undisturbed at 226·5 − 2·6 = 223·9 days. In this preliminary discussion the first three residuals (of the Harvard table) were disregarded; they are printed in italics as of inferior value. All others were included, the simple mean of several at one value of E being taken.

S Ursæ Majoris. - *Adopted Period* $226{\cdot}5 - 2{\cdot}6 = 223{\cdot}9$ *Days.*

E.	Harv. Res.	C_1.	$H-C_1$.	E.	Harv. Res.	C_1.	$H-C_1$.	$O-C_2$.
	d	d	d		d	d	d	d
-28	+73	+60·2	+13	0	+ 1	-12·6	+14	- 3
-26	+37	+55·0	-18	1	+ 5	-15·2	+20	+ 3
-18	-42	+34·2	-76	3	+ 2	-20·4	+22	+ 5
-11	+14	+16·0	- 2	6	-26	-28·2	+ 2	-15
-10	+14	+13·4	+ 1	8	-21	-33·4	+12	- 5
- 9	+15	+10·8	+ 4	9	-13	-36·0	+23	+ 6
- 8	+ 8	+ 8·2	0	10	- 8	-38·6	+31	+14
- 7	+ 6	+ 5·6	0	11	-31	-41·2	+10	- 7
- 6	+ 2	+ 3·0	- 1	12	-30	-43·8	+14	- 3
- 5	- 2	+ 0·4	- 2	13	-28	-46·4	+18	+ 1
- 4	+ 2	- 2·2	+ 4	14	-33	-49 0	+16	- 1
- 3	- 6	- 4·8	- 1	15	-69	-51·6	-17	-34
- 2	- 9	- 7·4	- 2	16	-70	-54·2	-16	-33
- 1	-10	-10·0	0					

The calculated values C_1 decrease at the rate of 2·6 per unit E, the initial value being chosen to suit the residuals from E = -18 to -1. The accordance of the first three is no worse than we might expect from the description of the observations. After E = -1 comes the marked change of epoch, and in the column $O-C_2$ new values are obtained by a simple change of 17 days. After E = 14 comes another marked change, though it is not very clear whether E = 15 and 16, which are discordant in any case, belong to the group preceding or following them. In *discussing* the next set they were omitted, though they are included in the comparison below.

S Ursæ Majoris. Period $226{\cdot}5 + 0{\cdot}4 = 226{\cdot}9$ *Days.*

E.	Harv. Res.	C_3.	$O-C_3$.	E.	Harv. Res.	C_3.	$O-C_3$.
	d	d	d		d	d	d
15	-69	-31·8	-37	23	-21	-28·6	+ 8
16	-70	-31·4	-39	24	-35	-28·2	- 7
17	-26	-31·0	+ 5	25	...	-27·8	...
18	-30	-30·6	+ 1	26	-40	-27·4	-13
19	-20	-30·2	+10	27	- 2	-27·0	+25
20	-27	-29·8	+ 3	28	-24	-26·6	+ 3
21	-38	-29·4	- 9	29	-28	-26·2	- 2
22	-57	-29·0	-28	31	+ 3	-25·4	+28

The residual at E = 31 is no larger than has already occurred in the series, but it is followed by others like it, whereas the preceding large errors are isolated, with the exception of those at E = 15 and 16, which are anomalous in any case.

S Ursæ Majoris. Period $226{\cdot}5 + 2{\cdot}9 = 229{\cdot}4$ *Days.*

E.	Harv. Res.	C_4.	$O-C_4$.	E.	Harv. Res.	C_4.	$O-C_4$.
	d	d	d		d	d	d
31	+ 3	+ 5·5	− 3	39	+34	+28·7	+5
32	+ 7	+ 8·4	− 1	40	+37	+31·6	+5
33	+12	+11·3	+ 1	41	+43	+34·5	+8
34	+18	+14·2	+ 4	42	+44	+37·4	+7
35	+14	+17·1	− 3	43	+39	+40·3	− 1
36	+20	+20·0	0	44	+38	+43·2	− 5
37	+23	+22·9	0	45	+38	+46·1	− 8
38	+16	+25·8	−10	46	+53	+49·0	+4

There is now a change of period (without a "jump") to 224·3 days, which seems to last to the end of the Harvard table. It will be seen that changes are to be expected about every 15 units of E, as at

$$(-15),\quad 0,\quad +15,\quad +30,\quad +45,\quad (+60),\quad (+75),$$

but those in brackets passed without event.

S Ursæ Majoris. Adopted Period $226{\cdot}5 - 2{\cdot}2 = 224{\cdot}3$ *Days.*

E.	Harv. Res.	C_5.	$O-C_5$.	E.	Harv. Res.	C_5.	$O-C_5$.
	d	d	d		d	d	d
45	+38	42·6	− 5	62	− 6	+ 5·2	−11
46	+53	40·4	+13	63	+11	+ 3·0	+ 8
47	+44	38·2	+ 6	64	+ 2	+ 0·8	+ 1
48	+39	36·0	+ 3	65	−10	− 1·4	− 9
49	+28	33·8	− 6	66	+ 4	− 3·6	+ 8
50	+30	31·6	− 2	67	−14	− 5·8	− 8
51	+33	29·4	+ 4	68	− 7	− 8·0	+ 1
52	+25	27·2	− 2	69	−20	−10·2	−10
53	+18	25·0	− 7	70	−19	−12·4	− 7
54	+13	22·8	−10	71	−10	−14·6	+ 5
55	+19	20·6	− 2	72	− 5	−16·8	+12
56	+15	18·4	− 3	73	−13	−19·0	+ 6
57	+17	16·2	+ 1	74	−25	−21·2	− 4
58	+14	14·0	0	75	−30	−23·4	− 7
59	+25	11·8	+13	76	−29	−25·6	− 3
60	+18	9·6	+ 8	77	−23	−27·8	+ 5
61	+ 7	7·4	0	78	−26	−30·0	+ 4

In all these cases the periods have been determined by a mechanical process to avoid bias. Thus for E = 61 to 75 the first and last give a difference of − 37 days for 14 periods, the next

inner pair − 19 days for 12 periods, and so on. The value of the period was adopted as

$$\frac{-37\times14-19\times12-24\times10-7\times8-0\times6-23\times4-6\times2}{14^2+12^2+10^2+8^2+6^2+4^2+2^2}=-2^{d}\cdot1 \text{ days.}$$

The value deduced from E = 46 to 60 by a similar process was − 2·2 days. Using these slopes with mean values as starting-points, the adjacent ends of the two series were in such close juxtaposition that there was no doubt about continuity over the break.

We have a jump of $+17^{d}$ before E = 0, a zero jump before E = 15, since the value of C_2 at E = 14 is − 49·0 + 17 = − 32 days, and the value of C_3 is − 32·2 for the same epoch.

At E = 29 the values of C_3 and C_4 are − 26·2 and − 0·3, a difference of + 26 days. At E = 44 we have $C_4 = +43\cdot2$ and $C_5 = +44\cdot8$, so that we may again suppose a zero jump. Calculating the exact values of E for which $C_2 = C_3$ and $C_4 = C_5$, we find 14·1 and 44·3, which gives an interval of 15·1 between jumps if we assume the intervals all equal. This places the series at − 1·0, 14·1, 29·2, 44·3, (59·4), 74·5, 89·6, or let us say at 00300, 03730, 07160, 10590, 14020, 17450, 20880, 24310. The last two Julian dates occur in 1916 and 1925, and it has therefore already been decided whether there was another change of period in 1916, though the *H.A.*, 55, records do not extend sufficiently far to determine the point.

As things stand we have, therefore, the following changes of period :—

E.	Period.		
	d	d	d
− 28 to 14	223·9		
15 to 29		226·9	
30 to 44			229·4
45 to 78	224·3		

This is a new form of oscillation; but we may reasonably assume for the present that the secular change is represented by the increase from $223^{d}\cdot9$ to $224^{d}\cdot3$. The change is small, but it is pretty certain; for it is fortunately attested by two long series of more than 30 cycles each. An error of 0·4 days per cycle would amount to 12 days in all, which the observations seem quite good enough to check. At least there can scarcely be a *decrease*.

But we now see how easily a decrease of period might be suspected. Had there been no observations prior to E = 15 (*i.e.* prior to 1869) the above figures would inevitably have suggested a secular decrease of period. Apparently we cannot be reasonably sure of a star until it has been observed over a considerable number of periods; and we may have to be content for the present with the evidence derived from the really well-observed stars.

R Serpentis 154615. *Adopted Period* 357·2 *Days.*

Two early maxima at E = −45 and E = 0 are omitted.

R Serpentis. Adopted Period 357·2 − 3·0 = 354·2 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
16	+43	+53	−10	30	+18	+11	+ 7
17	+47	+50	− 3	31	+10	+ 8	+ 2
26	+30	+23	+ 7	32	− 2	+ 5	− 7
27	+30	+20	+10	33	+ 1	+ 2	− 1
29	+26	+14	+12				

It seems possible that there is a break between E = 17 and E = 26; and if so, the period from E = 26 to E = 33 must be further shortened to 352·2 days: but for the present we leave things as above.

The period then changes to some value close to that adopted: with two conspicuous exceptions at E = 55 and 56, the Harvard residuals are all fairly close to −28 days, as below, in the column $H - C_1$.

	Harv.	$H-C_1$.				Harv.	$H-C_1$.	C_2.	$H-C_2$.
39	−29	− 1			64	−25	+ 3	+12	− 9
40	−32	− 4			68	−36	− 8	+ 4	−12
41	−32	− 4			69	−19	+ 9	+ 2	+ 7
42	−30	− 2			70	−35	− 7	0	− 7
43	−36	− 8			71	−21	+ 7	− 2	+ 9
44	−20	+ 8			72	−46	−18	− 4	−14
45	−27	+ 1			74	−22	+ 6	+ 8	− 2
46	−28	0			75	−33	− 5	+ 7	−12
47	−30	− 2			76	−19	+ 9	+ 6	+ 3
48	−28	0			77	−22	+ 6	+ 5	+ 1
49	−36	− 8			78	−14	+14	+ 4	+10
50	−39	−11			79	−32	− 4	+ 3	− 7
51	−26	+ 2			80	−25	+ 3	+ 2	+ 1
52	−22	+ 6	C_2.	$H-C_2$.	81	−24	+ 4	+ 1	+ 3
55	+ 7	+35	+27	+ 6	82	−30	− 2	0	− 2
56	+ 9	+37	+25	+12	83	−24	+ 4	− 1	+ 5
58	−23	+ 5	+21	−16					

We may elaborate the solution a little by assuming breaks at about every 18 cycles, say at 0·5, 18·5, 36·5, 54·5, and 72·5, in which case the suitable corrections to period would be, in days,

$$(?),\ -5,\ 0,\ -2,\ -1,$$

and the corresponding residuals are given in the column $H - C_2$.

In any case, a small increase is indicated.

T Hydræ 085008. Adopted Period 288·8 *Days.*

Omitting the first observation at E = −4, residual +24, we have

T Hydræ. Adopted Period 288·8 + 0·5 = 289·3 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
0	0	−1·2	+ 1	10	+ 3	+ 3·8	− 1
1	−5	−0·7	− 4	14	+ 9	+ 5·8	+3
4	−1	+0·8	− 2	15	+ 5	+ 6·3	− 1
5	+2	+1·3	+ 1	19	+ 8	+ 8·3	0
6	−3	+1·8	− 5	20	+ 6	+ 8·8	−3
9	−6	+5·3	−11	43	+21	+20·3	+1

The period then lengthens by about 2·5 days.

Adopted Period 288·8 + 3·0 *Days.*

47	− 5	− 3	−2	58	+38	+30	+ 8
48	+ 3	0	+3	59	+37	+33	+ 4
52	+12	+12	0	62	+15	+42	−27
53	+17	+15	+2	63	+62	+45	+17
57	+31	+27	+4				

The last two residuals are not very satisfactory, but do not upset the main conclusion.

R Camelop. 142584. *Adopted Period* +269·5 *Days.*

There seem to be breaks or jumps at every 9 cycles, not always taken ; say at

E = −0·5, (8·5), 17·5, 26·5, 35·5, (44·5), 53·5 ?

Adopted Period 269·5 − 4·0 = 265·5 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
− 6	−25	−28	+ 3	− 3	−20	−40	+20
− 5	−35	−32	− 3	− 2	−44	−44	0
− 4	−47	−36	−11	− 1	−47	−48	+ 1

Adopted Period 269·5 + 1·0 = 270·5 *Days.*

0	−72	−64	− 8	7	−60	−57	− 3
1	−63	−63	0	8	−65	−56	9
2	−57	−62	+ 5	13	50	51	+ 1
3	−66	−61	− 5	15	61	−49	−12
4	−43	−60	+17	16	−46	−48	+ 2
6	−51	−58	+ 7	17	−42	−47	+ 5

Adopted Period 269·5 − 4·0 = 265·5 *Days.*

E.	Harv.	Calc.	H−C.	E.	Harv.	Calc.	H−C.
18	−30	−38	+ 8	22	−50	−54	+ 4
19	−41	−42	+ 1	23	−63	−58	− 5
20	−50	−46	− 4	25	−61	−66	+ 5
21	−56	−50	− 6	26	−70	−70	0

Adopted Period 269·5 + 1·0 = 270·5 *Days.*

27	−48	−42	−6	32	−33	−37	+ 4
28	−35	−41	+6	33	−38	−36	− 2
29	−34	−40	+6	34	−41	−35	− 6
30	−40	−39	−1	35	−12	−34	+22
31	−45	−38	−7				

Adopted Period 269·5 + 1·5 = 271·0 *Days.*

36	+13	+10	+ 3	45	+30	+24	+ 6
37	+ 8	+12	− 4	46	+39	+25	+14
38	+ 9	+13	− 4	47	+28	+27	+ 1
39	+15	+15	0	48	+28	+28	0
40	+23	+16	+ 7	49	+36	+29	+ 7
41	+30	+18	+12	50	+38	+30	+ 8
42	+27	+19	+ 8	51	+17	+32	−15
43	+13	+21	− 8	52	+11	+33	−22
44	+25	+22	+ 3				

The last group is not very satisfactory, suggesting a break at E = 50·5, which would not fit in with the adopted uniform interval between breaks. It is not obvious how to readjust the interval to meet this possibility, and light is needed from later observations. But in any case the period of the last group seems larger than anything preceding, and would be further increased if we admit the break at E = 50·5.

R Persei 032335. *Adopted Period* 210·1 *Days.*

The changes are so small that it is perhaps treating them too seriously to give them in detail: but breaks are suggested at intervals of about 24 cycles, and the periods, mechanically calculated, came out

E.	Corrn. to Period.
(0) to 7	−2·2
8 to 31	−0·3
32 to 55	0·0
56 to (75)	+0·3

i.e. steadily increasing.

R Arietis 021024. *Adopted Period* 186·55 *Days.*

Here again the material is not large. Mechanical computations of the period give

E.	Corrn. to Period.	
− 3 to +20	0·0	alternative.
or −17 to +20	−0·3	
+26 to +49	+0·4	
+55 to +71	+0·8	alternative.
or +55 to +75	+0·2	

The question of a break at E = 72 can be decided by later observations; and it may be possible to infer whether there is one between −3 and −13. But in any case a slight increase of period is suggested.

R Vulpeculæ 205923. *Adopted Period* 136·8 *Days.*

Assuming breaks about E = −15·8, +41·8, and +99·4, we omit two isolated maxima at E = −17 and −16. Mechanical computation of periods then gives

E.	No. of Max.	Corrn. of Period. d
−10 to +40	28	−0·43
+42 to +99	24	+0·54
101 to 115	10	(0·0)

the last determination being uncertain. The evidence is consistent with an increase accompanied by an oscillation.

T' Herculis 180531. *Adopted Period* 165 *Days.*

There seem to be breaks at intervals of about 26·6 cycles, indicated by the values of C_1 below:—

Adopted Period 165 + 0·4 = 165·4 *Days.*

E.	H.	C_1.	H − C_1.	H − C_2.	E.	H.	C_1.	H − C_1.	H − C_2.
−26	−14	−1·0	−13		−5	+ 2	+ 7·4	−5	− 5
−22	+ 7	+0·6	+ 6	+ 3	−4	+ 3	+ 7·8	−5	− 4
−21	+ 2	+1·0	+ 1	−2	−3	+ 6	+ 8·2	−2	− 1
−20	+ 1	+1·4	0	−3	−2	+ 5	+ 8·6	−4	− 2
−19	+ 6	+1·8	+ 4	+2	0	+10	+ 9·4	+1	+ 3
−17	+ 6	+2·6	+ 3	+3	+3	+ 8	+10·6	−3	+ 1
15	+ 1	+3·4	− 2	−2	+4	+ 9	+11·0	−2	+ 2
−14	− 2	+3·8	− 6	−4	+5	+16	+11·4	+5	+ 9
− 6	+ 3	+7·0	4	+4	+6	+13	+11·8	+1	+ 6

Adopted Period 165 − 0·7 = 164·3 *Days.*

E.	H.	C_1.	H−C_1.	H−C_2.	E.	H.	C_1.	H−C_1.	H−C_2.
7	+ 6	+4·0	+ 2	− 1	25	− 6	− 8·6	+3	− 1
8	0	+3·3	− 3	−7	26	− 8	− 9·3	+1	− 2
9	+ 8	+2·6	+ 5	+1	27	− 5	−10·0	+5	+ 1
10	0	+1·9	− 2	−7	28	−20	−10·7	−9	0
13	0	−0·2	0	0	30	− 8	−12·1	+4	+10
14	+ 4	−0·9	+ 5	+4	32	−14	−13·5	0	

Adopted Period 165 + 1·0 = 166·0 *Days.*

34	−16	−15	−1	− 2	47	− 7	− 2	− 5	− 6
35	−11	−14	+3	+ 2	50	− 8	+ 1	− 9	−10
36	−11	−13	+2	+ 1	52	+ 9	+ 3	+ 6	+ 5
37	−15	−12	−3	− 4	53	+14	+ 4	+10	+ 9
38	−10	−11	+1	0	54	+ 5	+ 5	0	− 1
39	−10	−10	0	− 1	58	+ 5	+ 9	− 4	− 5
43	− 9	− 6	−3	− 4	59	+22	+10	+12	+11
45	+ 1	− 4	+5	+ 4					

Adopted Period 165 + 0·5 = 165·5 *Days.*

60	− 3	+1·7	−5	−2	78	+ 2	+10·7	− 9	− 5
61	+ 4	+2·2	+2	+4	79	+12	+11·2	+ 1	+ 5
63	0	+3·2	−3	−2	80	+ 7	+11·7	− 5	0
65	+ 2	+4·2	−2	−2	81	+ 6	+12·2	− 6	− 1
67	+ 8	+5·2	+3	+2	82	+ 7	+12·7	− 6	0
68	+ 8	+5·7	+2	+1	83	+ 8	+13·2	− 5	+ 1
72	+10	+7·7	+2	−1	84	+ 5	+13·7	− 9	− 2
74	+11	+8·7	+2	−2	85	+11	+14·2	− 3	+ 4
75	+16	+9·2	+7	+2	86	+30	+14·7	+15	+23
76	+ 2	+9·7	−8	−5	(87)	+ 3	+15·2	−12	− 4

There is a suggestion of a break between E = 75 and 76. If we admit it, then the period from 60–75 should be further increased—say to 166 days,—while that from 76 to 86 will remain much as at first. The breaks could then be placed at

$$-22{\cdot}2,\ -5{\cdot}6,\ +10{\cdot}7,\ +27{\cdot}0,\ (43{\cdot}3),\ 59{\cdot}6,\ 75{\cdot}9,\ 92{\cdot}2,$$

thus isolating the first observation at E = 26. The corrections to period would then be as follows:—

Hypothesis C_1.		Hypothesis C_2.	
		−22 to − 6	−0·2
−26 to + 6	+0·4	− 5 to +10	0·0
+ 7 to +32	−0·7	+10 to +27	−0·5
+34 to +59	+1·0	+28 to +59	+1·0
+60 to +86	+0·5	+60 to +75	+1·0
		+76 to +92	0·0

Either hypothesis suggests alternation with a slow secular increase.

Summary.

A number of stars in Phillips's Group I. have been examined, especially those for which there is already a long series of recorded maxima. The evidence is in favour of a secular increase of period, though the liability to oscillation in period tends to obscure the question of secular change. A few apparent exceptions are reserved for further scrutiny. The following details give a fair view of the situation, the quantities tabulated being suggested corrections to the period adopted in *H.A.*, **55**, in days:—

U Virginis: +0·7, −1·0, +1·5, −0·2, +2·0, +1·0. Oscillation with secular increase. Interval between changes, 14 × 207 days = 2900 days say.

S Cephei: −4·0, +7·0. One large increase.

T Sagittarii: 0·0, +14·0. One large increase, *but group not yet assigned.*

η Geminorum: −1·0, +1·0. *Group not yet assigned.* Has this star a double period like W Cygni? Seems well worth watching.

W Ophiuchi. Very few observations.

T Cassiopeiæ: −20·0, −3·0. One large increase.

S Ursæ Majoris: −2·6, +0·4, +2·9, −2·2. Terminal periods well determined. Oscillation with secular increase. Interval between changes, 3430 days.

R Serpentis: −3·0, 0·0; or alternatively −5, 0, −2, −1. In either case an increase of period.

T Hydræ: +0·5, +3·0. One definite increase.

R Camelop.: −4·0, +1·0, −4·0, +1·0, +1·5. Oscillation with secular increase. Interval between changes, 2430 days.

R Persei: −2·2, −0·3, 0·0, +0·3. Steady increase? Observations not numerous. Interval between changes, 5000 days?

R Arietis: 0·0 or −0·3, +0·4, +0·8 or +0·2. Increase? Observations not numerous. Interval between changes about 4000 days.

R Vulpeculæ: −0·4, +0·5, (0·0). Oscillation with secular increase? Last group uncertain. Interval between changes, 8000 days.

T Herculis: +0·4, −0·7, +1·0, +0·5; or alternatively −0·2, 0·0, −0·5, +1·0, 0·0. In either case oscillation with secular increase. Interval between changes, 26·6 × 165·5 = 4403 days, or alternatively 16·3 × 165·5 = 2698 days.

On the Nature of Short-Period Variables.

By H. C. Plummer.

1. The position of the problem presented by the variables of short period has recently been discussed by two writers. Both are adverse to what is known as the pulsation theory, and favour the normal interpretation of the spectrographic evidence. One of the writers, Father Hagen,* clearly understands that those who have studied the problem carefully may differ on fundamental points and yet are likely, dealing with the same set of facts, to have something in common. He therefore seeks the greatest common measure of their views. The other writer, Dr. Perrine,† is perhaps less familiar with the point of view involved in the pulsation theory, and his arguments are not quite so logical or so convincing as he appears to think. At the time when I first entertained the idea ‡ of a single pulsating star, the binary theory seemed to hold the field exclusively. It was necessary to insist on an alternative possibility. But now that the alternative has come actually into vogue in some quarters, it is easy to resume an attitude of impartiality and to see the difficulties involved in either position. But some of the objections taken to the hypothesis of a single star are not real.

2. In favour of the binary interpretation Dr. Perrine gives prominence to

"1. Representation of the observed variable line-displacements satisfactorily by orbital motion."

Now the pulsation theory has been suggested more or less independently at different times, and has been supported by arguments of very unequal cogency. But what led me to entertain the hypothesis in the first place was precisely that the representation by Keplerian motion is *not* always satisfactory. The discrepancies are notorious, and they are chiefly connected with variable stars. There was no attempt to evade the interpretation by Doppler's principle. On the contrary, this interpretation was perhaps pushed to a too literal extreme. Then difficulties arise, for in several cases the radial velocities are not compatible with pure elliptic motion. The anomalies have been assigned to epicyclic motions performed in some sub-multiple of the main period. But the explanation, if it can be called so, is far from convincing. It may be that the anomalies need not be taken too seriously. They are generally small, and it may not be necessary to interpret that part of the line-displacements which gives rise to them in terms of radial velocity. But to ignore them is impossible, and so long as the Doppler interpretation is applied exclusively, it is natural to look to some radial motion in the surface of the star either as an accessory to the orbital motion or as a complete substitute for it. The general resemblance between

* *A.N.*, 4995. † *Ap. J.*, 50, 81.
‡ *M.N.*, 73, 658, 665 ; 74, 662 ; 75, 573.

the velocity curve and the magnitude curve, though never perfect, may seem to confirm the binary hypothesis, but the very fidelity with which some minor feature of the one is reproduced in the other should rather be regarded as a suspicious circumstance.

3. Dr. Perrine refers to the character of the lines in the spectrum as a test of the pulsation hypothesis. But it seems very doubtful if the test can be regarded as decisive. The greater part of the evidence exists in the form of spectrograms taken with one-prism dispersion. Of much greater weight is the smaller number of spectrograms of the very few variable stars bright enough for a three-prism instrument. But among the latter is δ Cephei itself. It is only those spectra which show the greatest displacement from which any evidence on the point could be expected. As far as I know, they have not been specially examined from this point of view; but Dr. Moore, I believe, noted nothing very abnormal in his fine series of this star at the Lick Observatory. The evidence may then be taken to be negative. But is this surprising? The conditions may be examined. Let the star be contracting with a surface velocity V. Let P be the centre of the star's visible disc. The line-of-sight motion of a point at a distance θ from P will be $V\cos\theta$, and motions in the range $Vd(\cos\theta)$ will come from an area $\pi a^2 . d(\sin^2\theta)$. As a convenient form let us assume darkening towards the limb according to the law $1 - k\sin^2\theta$. Then the relative intensity of the light (or absorption) corresponding to a given value of θ will be proportional to $(1 - k\sin^2\theta)d(\sin^2\theta)/d(\cos\theta)$ or $(1 - k\sin^2\theta)\cos\theta$. Let this be y, and let x be the displacement of the corresponding part of the line, so that $x = \cos\theta$ on a certain scale. Hence the form of the intensity curve of the line will be represented by

$$y = x(1 - k + kx^2), \quad 0 < x < 1.$$

In the absence of any darkening at the limb, $k = 0$ and $y = x$, or the intensity curve is linear. The line has a sharp edge on the side of greatest displacement, and shades off uniformly to the undisplaced position. On the other hand, if darkening at the limb is complete, $k = 1$ and $y = x^3$, or the intensity curve is a cubical parabola. The precise form of this result depends on the assumed law of darkening, and is really arbitrary. But it is enough to show that the linear shading of a line may quite easily be sharply modified. The hard edge remains, but the undisplaced side will be practically cut off. There is also the effect of rotation, which will broaden the line on any hypothesis. All this applies to a monochromatic absorption. If lines of this character are placed together in that complex structure which constitutes a stellar spectrum of type F or G, there seems no reason to be surprised if the quality of the lines fails to give positive evidence in favour of the pulsation hypothesis. Naturally this point has been borne in mind from the beginning.*

* *Cf.* Eddington, *M.N.*, 79, 186; Adams and Shapley, *Obs.*, 42, 167, 168.

4. Another point which Dr. Perrine makes, and to which he attaches so much importance that he returns to it in a separate note,* is very difficult to understand. For some reason he considers it a necessary consequence of the pulsation hypothesis that the mean radial velocity of a group of stars with no systematic tendency to approach or recede should coincide with the maximum rate of approach ($-$K in the ordinary notation) inferred from the velocity curves. Apparently the mean value of $V = \gamma + K \sin nt$ is γ when V is interpreted on the orbital hypothesis, and $\gamma - K$ when V is interpreted on the theory of pulsations. If Dr. Perrine is thinking of the star expanding explosively from its most contracted state, he supposes a condition which can hardly be reconciled with a continuous velocity curve. This is *not* the theory of free oscillations in a single star.

For the amplitude of the oscillation Professor Eddington † uses the element $a \sin i$ of the supposed orbit. There seems no reason why the correct expression should not be used. The amplitude corresponds to half the projected width of the orbit measured between parallel tangents at right angles to the line of sight, and is therefore

$$\sqrt{(a^2 \sin^2 \omega + b^2 \cos^2 \omega)} \sin i = a \sin i \sqrt{(1 - e^2 \cos^2 \omega)}.$$

This is identical with $a \sin i$ when $e = 0$, which is not the case in general, and when $\omega = 90°$, the condition which is approximately fulfilled in most cases. The use of $a \sin i$ exaggerates the amplitude systematically, if only to a slight extent.

5. It has been generally recognised that the short-period variables are concentrated towards the plane of the Milky Way, a fact on which Dr. Perrine lays great stress. In such questions it seems desirable to have some standard of comparison by which the degree of concentration can be judged. For stars distributed uniformly over the sky it is obvious that one half will be found within 30° of the galactic plane. If θ is the polar distance of a star, the mean polar distance of a group is

$$\int_0^{\frac{1}{2}\pi} \theta \sin\theta \, d\theta \div \int_0^{\frac{1}{2}\pi} \sin\theta \, d\theta = \Big[\sin\theta - \theta\cos\theta \Big]_0^{\frac{1}{2}\pi} = 1,$$

or the mean galactic latitude for uniform distribution is about 33°. But the stars in general are not uniformly distributed, and it seems well to know the mean galactic latitude to be expected from a group of stars which occur with equal frequency among all the stars distributed as we find them to be. Now this distribution depends on the limiting magnitude. Taking it to be the eleventh magnitude (visual), and neglecting the want of symmetry between the two hemispheres, we have ‡ for the density

$$D = 21{\cdot}5 - 12{\cdot}7 \cos 2\theta + 5{\cdot}1 \cos 4\theta - 3{\cdot}9 \cos 6\theta.$$

* *Ap. J.*, 50, 148. † *M.N.*, 79, 6.
‡ *M.N.*, 78, 672.

Hence the mean polar distance is

$$\frac{\int_0^{\frac{1}{2}\pi} \theta(55{\cdot}7 \sin\theta - 17{\cdot}8 \sin 3\theta + 9{\cdot}0 \sin 5\theta - 3{\cdot}9 \sin 7\theta)d\theta}{\int_0^{\frac{1}{2}\pi} (55{\cdot}7 \sin\theta - 17{\cdot}8 \sin 3\theta + 9{\cdot}0 \sin 5\theta - 3{\cdot}9 \sin 7\theta)d\theta}$$

$$= \frac{55{\cdot}7 + 2{\cdot}0 + 0{\cdot}4 + 0{\cdot}1}{55{\cdot}7 - 5{\cdot}9 + 1{\cdot}8 - 0{\cdot}6} = \frac{58{\cdot}2}{51{\cdot}0} = 1{\cdot}14 = 65^\circ{\cdot}3$$

since

$$\int_0^{\frac{1}{2}\pi} \sin n\theta \, d\theta = \frac{1}{n}, \qquad \int_0^{\frac{1}{2}\pi} \theta \sin n\theta \, d\theta = \frac{\sin \frac{1}{2}n\pi}{n^2}$$

when n is odd. The mean galactic latitude of all the stars to the eleventh magnitude is therefore about 25°. For a brighter limiting magnitude the corresponding latitude will be slightly higher, for a fainter magnitude it will be lower. The median latitude, on the other hand, is given by

$$25{\cdot}5 = 55{\cdot}7 \cos\theta - 5{\cdot}9 \cos 3\theta + 1{\cdot}8 \cos 5\theta - 0{\cdot}6 \cos 7\theta$$

or $\theta = 71^\circ$. Thus half the stars in the sky to the eleventh magnitude, as actually distributed, are within about 19° of the galactic plane, a remark of some general importance when galactic distributions are in question.

6. When therefore it is found that the mean galactic latitude of 168 short-period variables is 16°·5, and that 88 of them are within 10° of the equatorial plane, there can be little doubt of the reality of the galactic connection. But it may easily be exaggerated by the effect of selection. In order to measure the magnitude of a variable accurately it is necessary to have suitable comparison stars in the surrounding field, and such stars are more likely to be found in rich than in poor regions. The same circumstance applies to the discovery of variables, especially of those with a range of a magnitude or less. Hence it would be expected that more short-period variables would be discovered in the rich galactic regions than in the sparse regions far from the Milky Way. A large proportion of these stars have been found at Harvard, and the methods of universal survey characteristic of that observatory would reduce the effect of selection, but might not completely overcome it. On the other hand, if an observer were to set out with the deliberate object of finding variables by photographing the sky, he would obtain effectively the greatest amount of material with a given labour at the telescope by selecting dense in preference to poor regions. There would be a natural tendency to neglect the latter, and an exaggerated appearance of galactic concentration might easily result. In a rather similar way a search for minor planets is naturally conducted in the neighbourhood of the ecliptic, with the result that a relative deficiency in the higher inclinations is to be expected.

7. Dr. Perrine has examined various relations to be found among

the different elements of the velocity curves and the light curves. They are for the most part rather vague, perhaps unavoidably so owing to the very small amount of relevant material. They are doubtless suggestive, and will probably repay further consideration. But two general remarks may perhaps be allowed. Dr. Perrine, interpreting such results from the binary point of view, finds evidence in them of the correctness of that view. But if they are interpreted in the light of the pulsation theory, they appear equally natural. For example, he finds high orbital velocity to be associated with greater light variation. In the language of the pulsation theory, this only means that the more violent the pulsation the greater the range of magnitude. Such observed relations are no doubt interesting, and may prove important, but they seem to help little towards a decision between the two theories. The other remark is concerned with a real and constant difficulty in correlation statistics, and it is particularly well illustrated here. This difficulty consists in making the right choice of independent variables to correlate. The velocity curve gives directly the period P, the projected orbital velocity K by its amplitude, and the eccentricity e and the "longitude of periastron" ω by its shape. The last element should have no special tendency: the apparently disconcerting thing in the case of variables is that it has. That leaves the three elements, P, K and e, of general significance, and no other in the absence of the second spectrum. Two other quantities,

$$a \sin i = C_1 . KP(1-e^2)^{\frac{1}{2}}, \quad m'^3(m+m')^{-2} \sin^3 i = C_2 . K^3 P(1-e^2)^{\frac{3}{2}},$$

where C_1, C_2 are constants depending on the units, are derived by computation. The ratio of the second to the first is represented by $K^2(1-e^2)$, and therefore when Dr. Perrine finds that large eccentricities correspond with small values of this ratio, he only finds what must necessarily be true unless K and e are correlated. This is a point which he reserves for separate discussion, but it illustrates what appears to be a difficulty in this line of attack. The velocity curve gives too little information, and that little in an entangled form.

8. On the whole it seems fair to conclude that the pulsation theory is consistent with the facts of observation, and that no direct phenomenal test is likely to decide against it. The quantities P, K, e and ω on this theory define the period, intensity and type of the pulsation wave, and any correlations which may appear are as natural as when interpreted according to the binary hypothesis. On the latter theory the observed relation of periastron to the line of nodes presents a difficulty at first sight which disappears entirely on the alternative assumption. The absence of eclipses, supported by the purely negative evidence of a single spectrum, needs no explanation when the system is reduced to a single star. Up to this point the pulsation theory has at least something in its favour and nothing very serious against it. The real test comes from another direction. It is necessary to explain the maintenance of

the pulsations consistently with the facts by a mechanism not at variance with dynamical theory. The theory of free pulsations has been worked out by Professor Eddington with great skill and the right degree of pertinacity. It requires a diminution of the period which is not borne out by observation. In particular, the observed change in the period of δ Cephei is negligible in comparison with that which the theory, in the absence of some unknown source of energy, demands. Mr. Jeans is more uncompromising in his conclusions, and insists with his great authority that the pulsations must be in the nature of forced and not free oscillations. He has suggested that the rotation of a single elongated body with critical velocity in a tidal field might give rise to periodic explosions and so explain the observed light curves. But the nature of the gravitational field requires to be specified, and, as he is fully aware, the hypothesis is difficult to reconcile with the evidence of the velocity curve.

9. These considerations naturally bring us back to the binary hypothesis, and lead us to ask whether its difficulties (or incongruities, as Professor Eddington calls them) are so serious as they may have appeared. Father Hagen thinks they are not, and supposes as an explanation of the variability of light that the smaller companion has the power of causing eruptions of light in the primary by the mere fact of approach. More precise details he thinks unnecessary, and this assumption is consistent with the forced oscillations which Mr. Jeans requires, and perhaps provides the prominence effect on a large scale which I suggested in 1913. For such difficulties as have been felt in regard to the orientation of the orbit in space, its considerable eccentricity and the relatively small mass of the companion, Father Hagen has one facile explanation. All these properties enhance the apparent light variation, and in their absence the star may vary, but so slightly as to escape discovery. This mode of escape from one difficulty immediately seems to suggest another. The range of variation usually found is not constant, but it approaches a fixed standard in the short-period variable much more closely than would be expected if the circumstances accidentally favouring discovery alone were concerned. Yet it is not easy to escape from Father Hagen's main assumption in some form. The two principal ideas by which it has been sought to explain the light variations involve the action of a resisting medium and some kind of eruption. If the latter be combined with a binary hypothesis, the chief effect is naturally connected with the periastron passage, and Father Hagen has so completely avoided all compromising detail that his suggestion may well serve as a starting-point for further consideration.

10. In spite of his discretion and brevity, one point can hardly be accepted without reserve. He asserts that, whatever the cause may be, the variable will be the more easily discovered the smaller the mass of the companion. This seems extremely improbable. Pushed to its extreme, a mere meteor would cause a large variation, and stars with the largest variation would hardly

be detected as spectroscopic binaries. But it is not merely improbable—it is almost certainly untrue. For we have

No. of stars.	2	8	6
$m'^3(m+m')^{-2}\sin^3 i$	·00006	·0012	·0045
Mag. range.	0·1	0·61	0·78

when the stars are arranged according to the mass function.

The first group is not significant, containing Polaris, the variability of which would not have been discovered if it had not been Polaris, and β Cephei, which is of B type. The number is so small that there is nothing improbable in supposing i small. But the two following groups are just large enough to give a clear indication, and as far as they go it is undeniable that Father Hagen's easy explanation is not only improbable in itself but simply fails on the facts. The value of the mass function gives a very indeterminate equation. On the assumption that $m+m'=1$, the above mean values give $m'\sin i = 1/26$, $1/9$, $1/6$ respectively. On the assumption that $m'=1$, they give $(m+m')\sin^{-3/2} i = 130, 29, 15$ respectively. It is possible that the truth lies nearer the second than the first assumption. The important thing to be noticed is that the ratio m'/m tends further from unity, with a given value of the mass function, the greater the mass of the system. In order to bring the ratio as near unity as visual double stars and ordinary spectroscopic binaries have shown to be usual, the mass of the system must be improbably small. Not that this evidence can be held to rule out very unequal divisions of mass. That from spectroscopic binaries rests on double spectra, which naturally come from nearly equal components. In the case of double stars a distinct difference of mass may imply so wide a contrast in brightness as to hinder discovery. Apparently we should consider the ratio of the masses in short-period variable stars on the binary hypothesis as of the order 10 : 1.

11. The question of "small orbits" is often misunderstood. It is really concerned with the relations between period and density. Let ρ, ρ' be the mean densities, b, b' the radii of the two stars, $a+a'$ the relative mean distance, and n the mean motion. Then

$$\tfrac{4}{3}\pi\gamma\rho b^3\left(1+\frac{m'}{m}\right) = n^2(a+a')^3$$

$$> n^2(b+b')^3(1-e)^{-3}$$

and therefore

$$\rho > \frac{3n^2}{4\pi\gamma}\left(1+\frac{b'}{b}\right)^3\left(1+\frac{m'}{m}\right)^{-1}\left(1-e\right)^{-3}.$$

Even when b'/b is small and m'/m not large, it is in general necessary that

$$\rho > 3n^2/4\pi\gamma\,(1-e)^3$$

if the stars are to remain out of contact. If the stars are equal, and $m = m'$, $b = b'$, this limiting value is four times higher. The lower limit of the mean density may be written

$$\rho > 0{\cdot}02 P^{-2}(1-e)^{-3}\,\text{gr./cm.}^3$$

when the period P is expressed in days. This gives $P\sqrt{\rho} > 0{\cdot}14$ or $P\sqrt{(54\rho)} > 1$ even when the eccentricity is neglected. On the average $e = 0{\cdot}3$, $(1-e)^{-3} = 3$, and $P\sqrt{(54\rho)} > 1{\cdot}7$. This value is about double that found by Professor Eddington, and an acceptance of the binary hypothesis must involve a considerable revision of his theory. Apart from this, there seems to be nothing impossible in the limit for variables with a period of more than one day. Thus for δ Cephei the limit is about 0·004, and $\rho = 0{\cdot}01$ should give clearance at periastron. With a radius of ten times the sun's radius the mass would be ten times the sun's, and with the sun's surface brightness the absolute magnitude would be five higher. These are reasonable figures, and Professor Eddington's theory may be capable of the necessary modification. For the merihemeral variables, if RR Lyræ (type A) may be taken as representative, $\rho > 0{\cdot}16$, and for circular orbits performed in four hours $\rho > 0{\cdot}72$. Of course small values of K or $a \sin i$ imply a relatively small companion, and so far from presenting a difficulty are favourable to the density of the main star not exceeding a reasonable limit.

12. An important question related to the period in the orbit is concerned with the period of the rotation of the main star. If ω is the angular velocity, equilibrium requires that

$$\rho > \omega^2/2\pi\gamma C,$$

where C is a constant depending on the constitution of the body. Hence if ρ is little in excess of the minimum compatible with the mean orbital motion n,

$$\omega^2/2\pi\gamma\, C < 3n^2/4\pi\gamma(1-e)^3$$

or

$$\omega^2/n^2 < 3C/2\,(1-e)^3.$$

Now if $C > 1$ equilibrium is impossible in any undivided figure, by Poincaré's well-known theorem. Practically the value of C is much lower, being 0·361 for Roche's model and about 0·31 for Mr. Jeans' adiabatic model. Thus we may take $C = 1/3$, so that

$$\omega^2/n^2 < \tfrac{1}{2}(1-e)^{-3}.$$

Hence ω and n can be equal if $e = 0{\cdot}2$ even when the density is at its lowest limit, and generally it will be considerably higher. Therefore considerations of rotational equilibrium in the principal star will not preclude equality between rotation and mean orbital motion, though they may point to a state very sensitive to disturbance by an outside body. But it will be interesting to consider the condition in which ω is considerably less than n. In fact, it is necessary to do so. The surface velocity of the sun has a range

of $\pm 2\cdot2$ km./sec. Since the period of δ Cephei is five times less than the equatorial rotation period of the sun, this assigns a total range of 22 km./sec. to the star, if equal in size to the sun. If then the dimensions of δ Cephei are ten times greater, as they may easily be, the range becomes 220 km./sec., corresponding to a width of 3·3 Å.U. in the spectrum lines at λ4500. This is no doubt subject to considerable discount on account of darkening at the limb, but it can hardly be reduced to the observed width 0·4 Å.U.* This is a much more definite point than the corresponding question suggested by the pulsation theory. Sharpness of line is characteristic of this class of stars, and it can scarcely be reconciled with a large star in rapid rotation. Either these stars are of the same dimensions as the sun, or their rotational velocity is small compared with their orbital motion. The second of these alternatives seems the more probable. The dilemma can be evaded if the axis is always directed towards the sun: but is this likely as a general rule?

13. For a few of the short-period variables the results of applying the hypothesis of galactic motion are available. They are as follows:—†

P.G.C.	H.R.	Star.	Type.	ϖ.	m_0.	m_0'.
5532	8238	β Cephei	B1	″·0047	1·7	4·6
325	424	Polaris	F8*p*	·0361	4·9	3·2
4493	6616	X Sagittarii	F8*p*	·0052	3·3	2·4
4564	6742	W ,,	F5*p*	·0079	4·2	2·3
5370	7988	T Vulpeculæ	Fo	·0091	5·6	3·0

No star of this class was discussed and failed to yield a plausible result, a fact entirely consistent with the the galactic relations of the group. Under m_0 is reproduced the absolute magnitude for parallax 0″·01, and under m_0' Mr. Shapley's corresponding values ‡ reduced to the same standard, which gives 10 for the sun. The latter are on the average 1·8 brighter, and this does not seem a very serious difference. The notable brightness of stars with spectra F*p* on the Draper classification, or with the characteristic *c* in Miss Maury's notation, was noted at the time that the results were obtained, and checked by a method independent of the galactic hypothesis. It is a fact obtained equally by others and now generally recognised. It is, however, disputed by Dr. Perrine, and there may be some interest in pointing out that it is also deduced on the hypothesis that the intrinsic motions are parallel to the Milky Way, for this hypothesis seems specially appropriate to the group of stars concerned, and it involves no other assumed relation between proper motion and distance.

14. Let us now consider Father Hagen's suggestion for a solution of the problem of variability. The companion S_2 is a relatively

* Adams, *loc. cit.*

† *M.N.*, 73, 183; 75, 123.

‡ *Ap. J.*, 48, 282.

small star describing an orbit (fig. 1) about the principal star S_1 with an eccentricity of about 0·3 on the average. As it passes periastron at P it has the effect of increasing the brightness of S_1 in the neighbourhood of the point B. This increase, which is presumably due to something in the nature of an eruption, is very local. Hence if the orbit were inverted, from the aspect of the sun, the outburst would take place equally but would escape observation. This explains why ω should always be near 90°, and never near 270°. At the same time, if S_2 is small relatively to S_1, the absence of eclipses of noticeable extent is easily understood, the more so as the maximum phase may be identified with the middle of the rapidly ascending branch of the light curve. At periastron the light

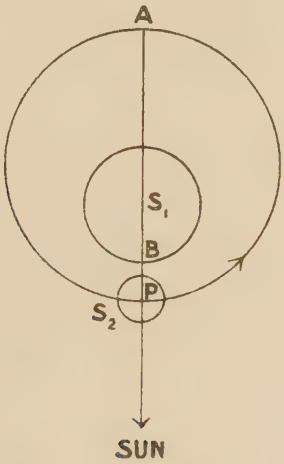


FIG. 1.

should be increasing most rapidly, and should reach its maximum about the time that the companion reaches the line of nodes, when the principal star is approaching the sun with maximum velocity. All this is fairly in agreement with the observed facts. But of course the most perfect explanation of the variability of the light —and Father Hagen expressly refrains from throwing any light on the nature of the outburst—is only a half solution of the problem confronting us. It is necessary not only to explain cases of observed variability, but to explain why, in cases not altogether dissimilar, a variation in the light is not observed. It is presumably for this reason that Father Hagen supposes that the effect of a small body may be more conspicuous than that of a large companion. But there is nothing in probability or in fact to support this hypothesis.

15. At periastron the two bodies may be supposed almost in contact. If b' be the radius of S_2, the purely local tidal effect at B will be proportional to m'/b'^3 or to ρ', the density of S_2. Thus

with a given mass m' any possible eclipse effect will be made smaller and the tidal effect larger by supposing the density of the companion very great. A small dense body may be more effective than a large body of low density. This consideration perhaps lends plausibility to Father Hagen's suggestion. But it is now necessary to develop the consequences of a remark made above.

There exists an ordinary class of spectroscopic binaries. These show no appreciable variation of light unless the geometrical conditions are such that they are at the same time eclipsing binaries. With increased sensitiveness in the methods employed for measuring the light the proportion of stars showing the second character has also increased. The cause of variation is geometrical, as opposed to physical, though with refined and careful observation a periastron effect has been noticed. This, however, is small and by itself would have escaped observation. The masses of the components are generally of the same order of magnitude. With very short periods the eccentricities are also small. The two bodies appear to rotate in a period identical with the period of the orbital motion. The facts seem consistent with the hypothesis that the two bodies have been derived by fissure from a single mass rotating in equilibrium.

The short-period variables, treated as spectroscopic binaries, differ in their properties from the class considered in nearly every possible way. Eclipses have never been observed. The masses of the components are very unequal. With short periods the eccentricities of the orbits are high. The narrow width of the lines in the spectra makes it difficult to believe that the rotation of the principal star synchronises with the very rapid orbital motion. All these differences taken together point very clearly to a radical difference of origin. The class of ordinary spectroscopic binary, including the eclipsing binary, may be derived from single bodies by fissure. The class to which the short-period variable belongs apparently contains binary systems in which the components have been brought together by the force of external circumstances. Such systems may be supposed to originate by capture, as opposed to division.

16. That two distinct classes of spectroscopic binary exist had already been inferred by Mr. S. Wicksell * from a purely statistical study of the periods and eccentricities. In the present case it was suggested by the difficulty of reconciling a spectrum possessing the *c* characteristic with rapid rotation. The bearing of the result, on which considerations of different kinds converge, seems obvious. The ordinary spectroscopic binary is a system in relative equilibrium as a whole originally, and is maintained in that state during the early stages of separation by tidal action. Therefore no great disturbance in its condition is to be expected. The other kind of binary starts with no approach to equilibrium as a complete system, though it tends to that state under tidal action. The eccentricity of the relative orbit is considerable, and a forced eruption coincid-

* *Meddelande från Lunds Astr. Obs.*, 63.

ing in time with the periastron passage seems quite natural. A good illustration of the value of Father Hagen's suggestion as to favourable conditions for observation and discovery is furnished by a_2 Geminorum. At first sight this star is highly anomalous, and is specially noted by Dr. Perrine. With a period of nine days and an eccentricity 0·5 it should be a short-period variable. On Father Hagen's principle the anomaly will disappear if ω is near 270°, for then the expected disturbance will be invisible from our point of view. As a matter of fact, it is at once verified that $\omega = 265°$. With a sufficiently rapid rotation of the apse a_2 Geminorum may yet become a variable of short period. Other instances, recently observed by Dr. Young,* are 1 Geminorum ($e = 0{\cdot}206$, $\omega = 203°$) and Boss 4669 ($e = 0{\cdot}468$, $\omega = 326°$). Both stars have periods of nine days, and are not known to be variable in spite of their unusually high eccentricities.

17. The ordinary equations for the variation of the elements give

$$\frac{da}{dt} = \frac{2a^2}{r}\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}, \qquad \frac{d\mathrm{P}}{dt} = \frac{3a\mathrm{P}}{r}\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}$$

$$\frac{dp}{dt} = 2r\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}$$

$$e\frac{de}{dt} = \left(\frac{p}{r} - \frac{r}{a}\right)\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}$$

$$e\frac{d\varpi}{dt} = \frac{\sin w(2 + e\cos w)}{1 + e\cos w}\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}$$

where p is the parameter and a the mean distance in the relative orbit, $\mu = m + m'$, w is the true anomaly, and T is the increase in the relative transversal acceleration, so that, if G is the tidal couple,†

$$\mathrm{T} = \frac{\mathrm{G}}{r}\left(\frac{1}{m} + \frac{1}{m'}\right).$$

Hence for the periastron and apastron distances it is easily found that

$$q_1 = a(1 - e), \qquad q_2 = a(1 + e)$$

$$\frac{1}{q_1}\cdot\frac{dq_1}{dt} = \frac{4r}{(1+e)p}\sin^2\tfrac{1}{2}w\left(1 + e\cos^2\tfrac{1}{2}w\right)\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}$$

$$\frac{1}{q_2}\cdot\frac{dq_2}{dt} = \frac{4r}{(1-e)p}\cos^2\tfrac{1}{2}w\left(1 - e\sin^2\tfrac{1}{2}w\right)\left(\frac{p}{\mu}\right)^{\frac{1}{2}}\mathrm{T}.$$

When the system as a whole is in relative equilibrium the tendency for the bodies to rotate faster owing to contraction is restrained by tidal action, and the corresponding reaction is in that

* *Popular Astronomy*, 28, 11.

† *Cf.* Jeans, *Problems of Cosmogony*, § 266.

sense which increases the orbital momentum. Therefore G and T are positive, and a, p and P increase with the time. The motion of the apse is periodic. Both the periastron and apastron distances increase with the time, but the latter much more quickly because T is greatest when $w = 0$. Hence the eccentricity also increases on the whole.

18. The maximum increase in the parameter is limited by the inequality

$$(1+\zeta)^{\frac{1}{2}}p^{\frac{1}{2}} < (1+x)(1+\zeta_0)^{\frac{1}{2}}p_0^{\frac{1}{2}}$$

which is imposed by the consideration that the total orbital momentum can never exceed its original value together with the proportion x of rotational momentum contained in the bodies. The factor $1+\zeta$ represents the increase in the force of attraction due to the deformation of the bodies from the spherical form, and falls rapidly from its maximum value $1+\zeta_0 = 1{\cdot}22$. Thus

$$p/p_0 < 1{\cdot}22(1+x)^2.$$

The original value p_0 may be identified with the radius a_0 of the circular orbit when the bodies rotate in contact, and since the periastron distance increases from the value a_0, though relatively slowly,

$$a(1-e) > a_0 > a(1-e^2)/1{\cdot}22(1+x)^2,$$

so that

$$1+e < 1{\cdot}22(1+x)^2.$$

When the divided masses are equal, $x = 0{\cdot}18$ and $e < 0{\cdot}71$. The eccentricity can only become parabolic when $x = 0{\cdot}3$, which requires a considerable disparity between the masses. With nearly equal masses,

$$\begin{aligned} a &< 1{\cdot}71a_0(1-e^2)^{-1} \\ \mathrm{P} &< [1{\cdot}71(1-e^2)^{-1}]^{\frac{3}{2}}\mathrm{P}_0\sqrt{(1{\cdot}22)} \\ &< 2{\cdot}36\mathrm{P}_0(1-e^2)^{-\frac{3}{2}}. \end{aligned}$$

Thus when the masses are equal the period cannot increase beyond 6·6 times its original value, though apparently a relatively small companion may be driven off and lost to the system entirely. Without external assistance the development of a double star out of a spectroscopic binary is in general impossible, as Russell and Jeans have shown. It is, however, unsafe to argue as to an upper limit to the eccentricity from determined double star orbits. These represent a very small fraction of the cases observed, and the extremely numerous 61 Cygni type might well be interpreted as examples of parabolic or even hyperbolic eccentricity. But with the masses divided as they are found to be the upper limit is set, for an isolated system, by the ratio of the parameter to the periastron distance.

19. When a new type of binary is contemplated, formed not by the division of one mass but by the accession of a second body

from outside, it may seem that a possible explanation of visual double stars is suggested. But this is difficult to believe owing to the observed association of types in these systems. The material * bearing on this question is collected in Table I., where the spectrum of the brighter star is shown on the left and that of the fainter on the top. It is immediately evident that in the majority of cases—130 out of 238—the spectra of both components are nearly the same. This fact certainly points to a common origin. Class A is clearly in excess, though not to the extent applying generally to all stars, Classes B, F, G, K are represented in very equal proportions in both components, and Class M is very rare. The evidence of the table seems distinctly against the fortuitous association of the components, and to that extent favours the explanation of Mr. Jeans, who attributes the properties of double-star orbits to the operation of equipartition of energy in a dense assemblage of stars.

Table I.

	B	A	F	G	K	M	
O	1	...	...	...	...	...	1
B	23	10	...	1	2	1	37
A	3	52	6	11	4	..	76
F	...	8	22	5	9	...	44
G	2	13	1	17	4	...	37
K	1	12	5	4	16	...	38
M	...	1	...	1	3	...	5
	30	96	34	39	38	1	238

The suggestion that our system has expanded from a previous condition of much greater density shows that the argument against the "island universe" theory, based on the supposed scale of the spiral nebulæ, is not in itself necessarily final.

20. When we return to the consideration of the different kind of system which has been imagined to represent the conditions of a short-period variable, the state of things is reversed. Instead of the principal star giving up its rotational momentum and increasing the dimensions of the relative orbit, the effect of tidal action will be to set the main body in rotation, or even to reverse an opposite tendency, at the expense of the orbital momentum. Hence in the formulæ of § 17 the sign of T is now negative. Consequently a, p and P will all decrease with the time. The periastron distance will decrease slowly, the apastron distance more rapidly, and the orbit will tend to become circular. The expected shortening of the period is important. For if the period of δ Cephei is decreasing, though at so slow a rate as to present a difficulty on

* *Harvard Annals*, 56, 231.

the pulsation theory, this fact would be at variance with the development of an isolated system in equilibrium on evolutionary lines. Systems ought to be found in which the period is increasing, and others in which the period is decreasing. The ultimate fate of the latter will depend on circumstances. As the main star acquires rotation the process will be accelerated as radiation causes it to contract. If the conditions are favourable its angular velocity will become equal to the mean orbital motion, and thereafter, relative equilibrium being established, the orbit will again expand at the expense of the rotational momentum acquired in the previous stage. But things may happen otherwise. Before the angular velocities become equal the periastron distance may become less than the sum of the radii of the stars. In other words, the process may come to an abrupt end in the precipitation of the companion on the main body. This possibility is the outcome of what seems to be a fairly strict line of reasoning. It points to a catastrophic change in the system, which can only be avoided if the companion is able to impart the necessary rotation to the larger star before the orbital momentum is reduced below the limit of safety.

21. At the present moment new facts are coming so fast that it is particularly unsafe to speculate on the subject of spectral relations. Yet there can be no harm if it is done with the full understanding that any scheme must be purely conjectural. If the Lockyer-Russell arrangement be accepted as a base, the position of the main classes of variable stars may be indicated provisionally as in fig. 2. The eclipsing binaries extend over the top of the curve. They are most frequently of types B and A, but apparently extend from W Crucis, a giant G star, to W Ursæ Majoris, a dwarf G star. Their position and range seem the least doubtful. Individually they should advance in type with lengthening period. The short-period variable is chiefly associated with the giant F and G types. As the type advances in the direction of A the period should become shorter, and the development into the cluster type of variable may be direct. A stage in which the period is a minimum is to be expected, and after this stage is past the period should lengthen and the variation should decline, if it has not already disappeared with the approach to equality between the periods of rotation and orbital revolution. It is then necessary to place the long-period variables mainly of type Md. The range of variation is generally so great that in a division according to luminosity the same star may almost be considered a giant at maximum and a dwarf at minimum. But the ascending branch seems clearly indicated by the bright hydrogen lines, and this is confirmed by the recently discovered affinity of the spectrum to the nebular and O types. That the long-period variables should be regarded as giants at maximum is also the conclusion of Mr. Gyllenberg * from a study of the proper motions. But this view is not altogether free from uncertainty, and it may be that the minimum should be considered the normal undisturbed magnitude.

* *Meddelande från Lunds Astr. Obs.*, 90.

22. It may be of interest to look at the spherical distribution of the long-period variables. This is shown in Table II. according to the equatorial system. The division is into equal areas, and the numbers in those regions through which the Milky Way passes are printed in bolder type. Several features of interest are immediately apparent. In R.A. there is the great deficiency at 11^h and the great excess at 20^h. In declination there is the marked deficiency near the equator, for which no explanation suggests itself. But it is the relation to the galaxy which is the more important. For the 240 areas into which the sky has been divided, the general average is 2·6 variables in each area. In the 41 areas through which the galaxy runs the corresponding average is 3·1. This is no great difference, and in fact, if the 11 galactic regions between 17^h and 21^h be excluded, the average for about

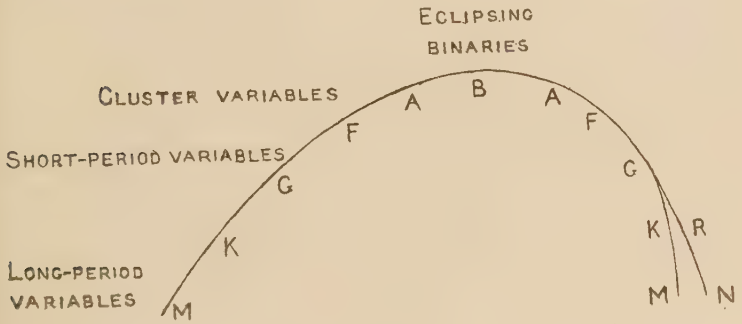


FIG. 2.

three-quarters of the Milky Way falls to the level of the whole sky. Even in this excluded locality the galaxy quite clearly steers a course between regions on either side where the variables are more frequent. The greatest number in a galactic region is 7, while there are six extra-galactic regions which contain more than 10. All the latter are between 15^h and 21^h and comprise the regions Libra-Scorpio-Ophiuchus, Sagittarius, Lyra and Delphinus. The course of the Milky Way, in its restricted geometrical sense, is between these constellations, and the variables may be connected with the outskirts of the great cloud masses which are a striking feature of this part of the sky.

23. When this distribution is compared with that of the short-period variables, which is the main purpose of this digression, a great difference is seen. It is unnecessary to reproduce the table corresponding to Table II. and like the latter based on material in *H.A.*, 56, because it is clearly a galactic distribution. Whereas the general average for the whole sky is 0·7 per area, the average for the 41 galactic areas is 2·1, or three times higher. Moreover, this average still remains as high as 1·7 when the galactic regions outside the exceptional area between 17^h and 21^h are alone considered.

The variables occur with the greatest frequency in the regions of Crux-Centaurus, Cepheus-Lacerta and Sagittarius-Aquila, which

TABLE II.

h	−90°	−53°·1	−36°·9	−23°·6	−11°·5	0°	+11°·5	+23°·6	+36°·9	+53°·1	+90°	Total
0	2	4	3	1	1	0	0	7	5	4		27
1	1	0	2	0	1	2	3	0	8	4		21
2	2	3	1	1	2	0	1	3	3	4		20
3	1	1	4	0	1	0	1	4	2	1		15
4	3	2	1	1	0	4	3	0	2	3		19
5	1	3	4	1	3	2	5	4	2	4		29
6	1	2	4	0	3	2	0	4	7	5		28
7	2	2	2	3	1	5	4	2	0	0		21
8	1	0	1	0	2	1	4	2	2	2		15
9	6	4	1	3	0	0	2	3	0	1		20
10	6	2	1	1	0	1	1	0	0	1		13
11	2	1	0	0	1	1	1	0	1	1		8
12	3	1	0	1	2	5	1	1	1	4		19
13	3	1	6	2	4	0	0	0	1	2		19
14	4	2	3	2	0	1	2	3	1	3		21
15	2	2	4	11	1	0	2	5	3	1		31
16	5	1	5	11	2	3	4	2	3	2		38
17	5	7	5	1	0	4	3	1	0	5		31
18	1	11	1	4	3	3	0	13	4	3		43
19	5	9	7	12	3	7	4	5	9	1		62
20	4	7	5	4	5	3	12	4	4	2		50
21	4	4	3	5	4	1	4	3	1	4		33
22	2	2	1	3	0	0	5	3	4	0		20
23	2	2	1	3	1	3	0	4	6	4		26
24												
Total	68	73	65	70	40	48	62	73	69	61		629

are galactic, and only to a less extent in Scorpio, Orion, Andromeda and Ursa Major, which are not galactic fields. Thus the distribution of the long-period and the short-period variables is very

different. One is struck once more with the wonderful richness of that part of the sky which lies between 15^h and 21^h R.A. and $\pm 53^\circ$ declination. The area is one-fifth of the whole sky, and the Milky Way runs directly through it. Wolf-Rayet stars occur to the extent of 31 per cent. Both short-period and long-period variables show a proportion of 35 per cent., but while the former for the most part lie close to the central line, the latter chiefly affect the edges of the Milky Way. Of eclipsing binaries 38 per cent. are here represented. There are also about 44 per cent. of

TABLE III.

Period and Spectrum (Md).

d	1	2	3	4	5	6	7	8	9	10	Total	I.	II.
50			1								1		
100			2		2	3					7	2	
150			3	4	5	1	1				14	3	
200			7	7	10	8	4	3			39	4	2
250		1	4	12	11	7	9	5	1	1	51	4	5
300	1		1	5	6	15	9	11	6	1	55	4	9
350	1		1	1	1	3	1	12	9	4	33	3	10
400		1	2			5	1	8	3		20	2	5
450						1		1	1		3		1
500			1	1	1		1				4		1
550						1					1		
600	1										1		1
650													
Total	3	2	22	30	36	44	26	40	20	6	229		
I.	1	0	5	4	2	3	0	4	2	1		22	
II.	1	1	3	0	3	6	3	13	3	1			34

globular clusters (according to Professor Bailey's list) and the same proportion of gaseous nebulæ. A short list of stars of β Lyræ type contributes no less than 60 per cent. to this region, and up to 1910 nearly the same proportion of novæ had been observed in the same area. Apart from possible cases in nebulæ, recent occurrences can hardly have reduced this ratio. The relative poverty in most respects of the opposite part of the Milky Way is scarcely less remarkable.

24. The majority of long-period variables belong to the spectral type Md, divided by Mrs. Fleming into ten grades, 1–10. The relation between these grades and the periods is shown in Table III., from which it is seen that, exceptionally large periods apart, the

period increases with the number of the grade. The table is bordered on the right and at the bottom with the numbers of stars in Mr. Phillips's Groups I. and II., distributed according to period and grade of spectrum. These numbers are small but interesting. In period the Group I. stars are distributed quite symmetrically about 275 days, the Group II. stars about 350 days. This clearly points to a genuine resolution into two distinct classes, depending on the shape of the magnitude curves, and it is not surprising that Mr. Gyllenberg has found with additional material that the unresolved distribution of the periods is not symmetrical. When the distribution of the two groups according to spectrum is considered, the preponderance of Md 8 in Group II. and of Md 3 and 4 in Group I. is noticed. The numbers are small and the grade of spectrum itself is variable, but there is a clear suggestion that the higher grades are associated with Group II. and the lower with Group I. If Professor Turner's suggestion is substantiated, that the order of evolution is from Group II. to Group I., this means that the spectrum changes in the direction Md 10 to Md 1, or the absorption spectrum alone in the sense Mc to Ma. This presumably indicates that the long-period variables should be assigned to the ascending branch of the curve of spectra. Slight and uncertain as the evidence is, the point seems to deserve further investigation by all possible means.

25. Mr. Gyllenberg has found no relation between the periods and the galactic latitudes of the long-period variables. Professor Turner, on the contrary, finds that stars of very long period affect the Milky Way. To examine this discrepancy Table IV., based on the Harvard list, has been constructed to show the relation between period and galactic latitude. Incidentally it is seen that the number (85) of stars in the galactic zone is sensibly smaller than the number (114) in the contiguous zones $\pm(10^\circ - 20^\circ)$ and even smaller than the number (87) in the zones $\pm(20^\circ - 30^\circ)$. This deficiency already noticed in the galactic zone may be due to extinction in space, as often suggested in similar cases. But it may also be that the missing variables are represented by other bodies which evade the enumeration because they are in an earlier or later stage of development. In regard to the periods, it is only evident that outside the immediate galactic zone a general mean of about 290 days rules, while in it the mean period is perhaps 50 days longer. This seems hardly consistent with Professor Turner's results,* which are in the same direction but point to a much more marked effect. In Table V. the material in Table IV. is combined for symmetrical zones and compared with the mean periods derived from the data for Groups I. and II. used by him. The effect of selection on the results is curiously marked within 30° of the galaxy. No doubt Mr. Phillips selected stars with the best determined and most regular periods. They are mainly of type Md, but this applies also to the general list. For some reason, which is not apparent,

* *M.N.*, **79**, 371.

all the twelve stars selected in the galactic zone have periods over 320 days, though the zone contains nearly as many stars with shorter periods. The excess of 100 days in the periods of the

TABLE IV.

Period and Galactic Latitude.

d	90	−60°	−30°	−20°	−10°	0°	+10°	+20°	+30°	+60°	+90	Total
50												
		1	1	1	1			1	1			6
100												
		2	2	6	3		2		1	1		17
150												
	1	8	1	4	2	1	4	6	3			30
200												
	4	21	8	8	7	4	11	7	11	3		84
250												
	8	12	10	16	6	6	9	11	14	5		97
300												
	3	15	10	12	8	4	13	11	16	1		93
350												
	4	9	5	8	9	9	8	5	9	2		68
400												
		7	5	3	6	8	3	1	2	1		36
450												
		1	1	1	1	4	3		2			13
500												
				2	2	1		2				7
550												
					1	1						2
600												
						1						1
650												
Total	20	76	43	61	46	39	53	44	59	13		454
Mean period	287	282	296	269	318	370	299	288	295	283		297 days.

galactic stars in Groups I. and II. has its origin therefore in selection, and the reason seems to require some elucidation before the marked relation between period and galactic latitude, especially in

TABLE V.

Gal. Lat. ±	454 Stars.		Group I.		Group II.	
	No.	Period.	No.	Period.	No.	Period.
0° − 10°	85	342d	4	448d	8	432d
10 − 20	114	283	5	310	11	339
20 − 30	87	292	7	312	4	365
30 − 60	135	288	9	241	22	324
60 − 90	33	286	4	229	6	321

Group I., can receive entire confidence. At the same time the more general material does bear out, if in a lower degree, Professor Turner's statement that the stars of very long period affect the Milky Way.

26. Though long-period and short-period variability must be con-

sidered together as parts of a larger problem, a direct connection between the two classes is not to be inferred. There is undoubtedly a general resemblance between the light curves of the short-period class and in particular of the long-period variables of Group II., and this is placed in a more definite form by the results of periodic analysis and the common phase rule. At the outside this may point to a general similarity in the growth and subsequent decay of a prominence effect in different circumstances. That there is any direct transition from the one class to the other is opposed by the very different distribution in space of the two kinds of stars. Moreover, though spectroscopic observations of long-period variables have been made hitherto under difficulties and the evidence is by no means complete, there is nothing to suggest that these stars are spectroscopic binaries. On the other hand, the range of velocity may be expected to be low with a long period and a relatively small companion, and the argument based on the irregularity of variation may not be conclusive because, for that matter, free oscillations ought to be more nearly isochronous than forced ones. The absence of the binary character, if definitely proved, is unfortunate, because the history of development suggested above for the short-period variable would fit in remarkably well with the results of Professor Turner's recent investigations of long-period variables. Thus the stage of approach of the two bodies following capture would be identified with the stars of Group II. with decreasing periods. This is succeeded by the transition state of minimum period, when equality is established between the time of rotation and the orbital period. The third stage, in which the bodies recede from one another and the variation dies away, is then represented by stars of Group I. with increasing periods. One might go further, and see in the precipitation of the captured body on the surface of the main star, before equality of period had set in, the origin of a nova. Such an event is a possibility in the circumstances supposed, and it would provide the grazing incidence postulated by some writers on the subject, and that with a much higher frequency than could be expected from direct encounters. But here the problem becomes too speculative for profitable discussion. The supposed facts do not encourage it, and new information, now rapidly accumulating, is liable to controvert any preconceived view. But it is interesting to observe how the general scope of the problem is widening. For example, there are the spectroscopic binaries of early type which give stationary H and K lines. The presumption is that the system in such cases is enclosed in a common envelope of calcium (and sodium). There should be stars not recognised as binaries with similar envelopes, but clearly they cannot be detected by the same test. Now it has been accepted by Sir N. Lockyer and by Pickering that the characteristic absence of the bright Hϵ line from the Md spectrum is due to absorption by the calcium H line. Is the higher calcium layer in the case of the long-period variable analogous to the calcium envelope of the binary system, and if so to what conditions is its

equilibrium in an elevated position due? Perhaps the study of calcium prominences on the lines of Mr. Pettit's paper * will contribute to the answer. The behaviour of calcium is also of interest for short-period variables, since in class F5 p = XII.c the calcium bands are narrower than in class XII., while in class F8 p = XIII.c they are wider than in any other stellar type. The continued and systematic observation of a few selected spectroscopic binaries is to be recommended, in order that the secular variations of the orbital elements may be determined.

27. Some points in a rather discursive paper may now be summarised:

(1) The variation of radial velocity in spectroscopic binaries, especially in short-period variables, is not always according to pure Keplerian motion in an elliptic orbit.

(2) The character of the spectrum lines is not likely to give decisive evidence against the pulsation theory. The paragraph relative to this point (§ 3) was written before the references there given had been consulted, and before a more recent note by Messrs. Shapley and Nicholson † came to hand. But it may stand as it was written.

(3) The theory of free oscillations would not entail, as Dr. Perrine supposes, a mean velocity of approach to the sun.

(4) The mean galactic latitude of all the stars to the eleventh magnitude is about 25°, and half the stars are within 19° of the galactic plane. The concentration of the short-period variables towards this plane is certainly far higher in spite of any possible circumstances favouring discovery in the denser regions of the sky. [The galactic concentration of the long-period variables is by this test slightly less than that of the stars to the limit named.]

(5) Correlations observed among the orbital elements are not likely to be decisive against the pulsation hypothesis, partly on account of the involved character of the relations, and partly because they are equally susceptible of interpretation on either theory.

(6) The real test comes not from the superficial phenomena but from the requirements of dynamical theory, which are unfavourable to the hypothesis of free oscillations.

(7) It is necessary to reconsider the binary hypothesis, and the form in which it has been stated by Father Hagen evades some of the difficulties which have been felt, though it is difficult to accept his suggestion that the smallness of the companion should, or in fact does, favour the discovery of variability.

(8) Short periods impose a lower limit on the density of the primary. This limit may require some revision in current theories of stellar equilibrium, but may not be impossible. The limiting density required by the orbital conditions will in general secure the rotational stability of the primary.

(9) If short-period variables are large bodies, as they appear to

* *Ap. J.*, 50, 206.

† *Proc. National Acad. of Sciences, U.S.A.*, 5, 417.

be, the characteristic sharpness of the lines in their spectra is evidence against rapid rotation. Hence it is inferred that these systems are not in rotational equilibrium as a whole, and differ in nature and origin from those spectroscopic binaries which in suitable geometrical circumstances appear as eclipsing binaries.

(10) In the latter type of binary the effect of tidal action is to check the tendency to more rapid rotation in the bodies and to increase the orbital elements a, e, P, but not beyond a limit which is distinctly restricted unless the masses are very unequal. It is well known that this fact requires some other agency for the evolution of double stars, but the prevailing similarity of type of spectrum even in wide pairs points to a common origin for the components. The rarity of class M stars in these systems is remarkable.

(11) In the system imagined to represent a short-period variable the effect of tidal action is, on the contrary, to impart rotation to the primary and to diminish the orbital elements a, e, P until equality is established between the periods of rotational and orbital revolution, after which the process is reversed unless the bodies have come into contact before the minimum period is reached.

(12) Among variable stars of all classes the position of the long-period variables alone seems uncertain. Their small proper motions point to great distance, but their distribution in space suggests a doubt. If all red stars are variable, the dwarfs may be irregular. The more regular stars (chiefly of type Md) may be giants, but a classification according to luminosity must depend largely on the choice between maximum and minimum as the normal magnitude.

(13) The spherical distribution of the long-period variables is examined in detail and compared with that of the short-period variables, which is very different.

(14) The periods of the stars in Groups I. and II. of Mr. Phillips are distributed symmetrically about 275 and 350 days respectively. In Mrs. Fleming's classification those of Group I. are chiefly in the grades Md 3–6, those of Group II. in the grades Md 6–9. This may be taken in connection with Professor Turner's suggestion that the order of evolution is from Group II. to Group I.

(15) Professor Turner's remark that stars of long period affect the Milky Way is confirmed by the examination of more extensive material, but not at all in the same degree. The stars chosen by Mr. Phillips show the effect of selection, the nature of which it would be well to know.

(16) Distribution and other facts make it impossible to suppose any direct connection between the variables of long and short periods, in spite of resemblances in the magnitude curves. The conditions summarised under (11) above would account for recent inferences as to the development of long-period variables, but are ruled out until these stars are shown to be binary systems.

The Spectra of Nova Aquilæ No. 3. II. *Transition to Nebular Spectrum.* By Joseph Lunt, D.Sc. (Plate 5.)

(*Communicated by S. S. Hough, M.A., F.R.S., H.M. Astronomer at the Cape.*)

The previous paper * dealt with a definite short-lived stage of the Nova Spectrum (June 10, 11, and 12) characterised by the appearance of a marked dark-line spectrum—mainly due to enhanced metallic lines—and the beginnings of a bright-band spectrum.

The measures were confined to plates taken on June 11 and 12, as on those dates the lines had become much sharper than on June 10. As the lines show a progressive shift towards the violet, measures of lines selected for their comparative sharpness in the spectrum of June 10 are here given to complete the record of the velocity displacements for the three days.

TABLE I.

Nova Aquilæ. 1918 June 10. Plate 5260.

First Set.

Measured λ.	Origin.	λ in ⊙.	Shift in λ.	Factor km./s. per λ.	Shift in km./s.	Corrn. for mean shift in λ.	Corrected λ.	O−T λ.
4082·50	$H\delta^1$	4102·00	−19·50	73·10	−1419	+19·63	4102·13	+ ·13
4319·58	$H\gamma^1$	4340·63	21·05	69·08	1454	20·77	4340·35	− ·28
4422·81	Ti_E	4443·98	21·17	67·84	1436	21·15	4443·96	− ·02
4448·13	Ti_E / He	4468·66 / 4471·65	22·03	67·08	1478	21·39	4469·52	− ·64
4460·24	Mg_E	4481·40	21·16	66·91	1416	21·45	4481·69	+ ·29
4550·23	Ti_E	4572·16	21·93	65·58	1438	21·88	4572·11	− ·05
4561·83	Fe_E	4584·02	22·19	65·41	1451	21·94	4583·77	− ·25
4528·44	$Fe_E\ Ti_E$	4549·73	21·29	65·91	1403	21·77	4550·21	+ ·48
4550·45	Ti_E	4572·16	21·71	65·58	1424	21·88	4572·33	+ ·17
4562·05	Fe_E	4584·02	21·97	65·41	1437	21·94	4583 99	− ·03
4739·95	Ti_E	4764·11	24·16	62·94	1521	22·80	4762·75	−1·36
4756·83	Ti_E	4780·17	23·34	62·73	1464	22·88	4779·71	− ·46
4782·80	Ti_E	4805·29	22·49	62·40	1403	23·00	4805·80	+ ·51
4800·70	Cr_E	4824·33	23·63	62·16	1469	23·09	4823·79	− ·54
4839·00	$H\beta^1$	4861·53	22·53	61·68	1390	23·27	4862·27	+ ·74
4900·5	Fe_E	4924·1	23·6	60·90	1437	23·56	4924·1	·0
4995·8	Fe_E	5018·6	−22·8	59·75	−1362	24·02	5019·8	+1·2

Mean shift −1435

* *M.N.*, 79, 416.

TABLE I.—*continued.*

Second Set.

Measured λ.	Origin.	λ in ⊙.	Shift in λ.	Factor km./s. per λ.	Shift in km./s.	Corrn. for mean shift in λ.	Corrected λ.	O−T. λ.
4072·09	$H\delta^2$	4102·00	−29·91	73·10	−2180	+30·01	4102·10	+ ·10
4308·69	$H\gamma^2$	4340·63	31·94	69·08	2206	31·76	4340·45	− ·18
4825·81	$H\beta^2$	4861·53	35·72	61·68	2203	35·57	4861·38	− ·15
4888·0	Fe_E	4924·1	36·1	60·90	2198	36·02	4924·0	− ·1
4982·1	Fe_E	5018·6	36·5	59·75	2181	36·72	5018·7	+ ·1

Mean shift −2194

Zero Set.

3933·53	K	3933·83	−0·30	76·23	−22·9	+0·32	3933·85	+ ·02
3968·29	H	3968·63	0·34	75·56	−25·7	0·32	3968·61	− ·02

Mean shift −24·3

	First Set.	Second Set.	Zero Set.
Mean shift	−1435	−2194	−24·3
Correction to ⊙	+ 10·5	+ 10·5	+10·5
Radial velocity	−1424·5	−2183·5	−13·8
Correction for solar motion	+ 17·1	+ 17·1	+17·1
Corrected radial velocity	−1407 km./s.	−2166 km./s.	+ 3·3 km./s.

Between June 12 and 15 a great change had taken place in the spectrum. The bright bands H_β, H_γ, H_δ, H_ϵ had become more prominent, and the continuous spectrum relatively weaker, and the spectrum may be regarded as in a transition stage between the enhanced dark-line and the bright nebular band spectrum.

Most of the prominent dark lines seen on the 12th—described in the former paper—had vanished, or left mere traces of absorption. The bright portion of H_β had widened in both directions and covered the dark lines $H\beta^1$ and Fe_E^2, but H_β^2 and Fe_E^1 are still prominent, whilst another prominent line, $H\beta^3$, had appeared still further to the violet of H_β^2. (See Plate.)

Cortie * finds a set of eight lines corresponding to $H\beta^3$ from H_β to H_ι displaced in the mean −2716 km./s. and Newall † records three, $H\beta^3$, $H\gamma^3$, and $H\delta^3$, displaced −2797.

In the Cape photograph, plate 5272, June 15, only one of these lines is prominent, viz. $H\beta^3$. $H\gamma^3$ is not prominent, but is measurable, and the mean of the two displacements for $H\beta^3$ and $H\gamma^3$ is −2782 (uncorrected to sun). $H\delta^3$ is not seen, and $H\epsilon^3$, if present with the same displacement as $H\beta^3$ and $H\gamma^3$, would fall on the fine K line previously recorded. $H\epsilon^3$ displaced −2782 km./s. would become λ3933.35, almost coincident with the fine K line measured at λ3933.58.

The second set of hydrogen lines displaced −2285 km./s. has

* *M.N.*, 79, 172. † *M.N.*, 79, 32.

now become much sharper and narrower, and next day (16th) has almost disappeared. The titanium, magnesium, and helium lines so prominent on the 12th have disappeared, but a few enhanced iron lines remain belonging to the first set (displaced − 1661), but none of the second set could be detected.

The broad bright bands are not uniform in intensity throughout their breadth, but show broad and ill-defined maxima and minima, best seen in H_β, which shows four bright maxima. The λ4640 group is beginning to differentiate itself as a broad bright band, but is still too ill-defined for measurement, as it fades off gradually as far as the iron line λ4737. Between the iron lines 4495 and 4679 the spectrum is brighter than the more refrangible region and is crossed by a number of rather broad, ill-defined maxima and minima which could not be interpreted. The measures indicate that the air group 4591–4705 is absent. The air lines were plotted with displacements − 1661 and − 2285, but neither coincided with the diffuse lines in the Nova Spectrum.

Table II. gives details of the measures of plate 5272 taken on June 15.

TABLE II.

Plate 5272. 1918 June 15.

First Set.

Measured in Nova λ.	Origin.	Rowland's λ in ☉.	Shift in λ.	Factor km./s. per λ.	Shift in km./s.	Corrn. for mean shift.	Corrected observed λ.	Observed minus Tabular λ.
3912·00	Ca	3933·83	− 21·83	76·23	− 1664	+ 21·79	3933·79	− 0·04
H_ϵ^1 3947·69$_2$	H,Ca	3969·40	21·71	75·54	1640	21·99	3969·68	+ ·28
H_δ^1 4079·00$_2$	H	4102·00	23·00	73·10	1681	22·72	4101·72	− ·28
4210·04	Fe_E	4233·33	23·29	70·83	1650	23·45	4233·49	+ ·16
H_γ^1 4316·22$_3$	H	4340·63	24·41	69·08	1686	24·04	4340·26	− ·37
4490·73	Fe_E	4515·51	24·78	66·40	1645	25·02	4515·75	+ ·24
4524·22	Fe_E	4549·64	25·42	65·91	1675	25·20	4549·42	+ ·22
4558·63	Fe_E	4584·02	25·39	65·41	1661	25·39	4584·02	·00
H_β^1 bright	H	4861·5	...	...	...			
4896·8$_2$	Fe_E	4924·1	27·3	60·90	1663	27·3	4924·1	·0
4991·0$_2$	Fe_E	5018·6	27·6	59·75	1649	27·8	5018·8	+ ·2
		Mean shift, 10 lines.			− 1661			

Second Set.

H_ϵ^2 3939·95$_2$	H	3970·18	− 30·23	75·53	− 2283	+ 30·25	3970·20	+ ·2
H_δ^2 4070·73$_2$	H	4102·00	31·27	73·10	2286	31·26	4101·99	− ·01
H_γ^2 4307·60	H	4340·63	33·03	69·08	2282	33·07	4340·67	+ ·04
H_β^2 4824·4	H	4861·5	37·1	61·68	2288	37·05	4861·45	− ·05
Bright	Fe_E	4924·1			...			
Invisible	Fe_E	5018·6			...			
		Mean shift, 4 lines.			− 2285			

TABLE II.—*continued.*

Third Set.

Measured in Nova λ.	Origin.	Rowland's λ in ☉.	Shift in λ.	Factor km./s. per λ.	Shift in km./s.	Corrn. for mean shift.	Corrected observed λ.	Observed minus Tabular λ.
$H\gamma^3$ 4299·84	H	4340·63	−40·79	69·08	−2818	+40·27	4340·11	− ·52
$H\beta^3$ 4817·0	H	4861·5	44·5	61·68	2745	45·1	4862·1	+ ·5
		Mean shift, 2 lines.			−2782			

Zero Set.

K	3933·58	Ca	3933·83	− 0·25	76·23	−19·1	+ 0·27	3933·85	+0·02
H	3968·33	Ca	3968·63	− 0·30	75·56	22·7	0·28	3998·61	−0·02
			Mean shift, 2 lines.			−20·9			

	First Set.	Second Set.	Third Set.	Zero Set.
Mean shift . . .	−1661	−2285	−2782	−20·9
Corrn. to sun . . .	+ 8·5	+ 8·5	+ 8·5	+ 8·5
Radial velocity . .	−1652·5	−2276·5	−2773·5	−12·4
Corrn. for solar motion .	+ 17·1	+ 17·1	+ 17·1	+17·1
Corrd. radial velocity .	−1635	−2259	−2756	+ 4·7 km./s.
Newall (*M.N.*, 79, 32) .	−1691	−2291	−2797	
Cortie (*M.N.*, 79, 172) .	−1761	−2415	−2716	

Summarising, we have the following velocity displacements corrected to sun :—

	Zero Set.	First Set.	Δ	Second Set.	Δ	Third Set.
June 10	−13·8	−1425	70	−2184	10	
11	−14·6	−1495	35	−2194	17	
12	− 9·1	−1530	123	−2211	66	
15	−12·4	−1653		−2277		−2774
Mean	−12·5	Range	228	Range	93	

It is important to notice that the first and second sets of lines move independently of each other, the progress of the lines of the first set being more rapid than that of the second set. This is particularly noticeable in the hydrogen lines, those of the first set moving closer to the lines of the second set.

On June 17 the nebular stage may be said to have commenced, but the chief nebular line does not appear until ten days later, and the 4363 band is not prominent before July 10.

The bright bands remain wide throughout the first period of observation, which extended to 1918 October 4, when observations were suspended until the Nova reappeared in the morning sky.

The mean widths of the bands in A° may be expressed approximately as $\lambda \times 0{\cdot}01104$, *i.e.* they are proportional to wave-length.

Widths of Bands.

Band.	λ	Calculated. λ×0·01104.	Observed Width.	O−C.
H_δ	4102·0	45·3	44·2	−1·1
H_γ	4340·6	47·9	46·5	−1·4
Neb. band	4363·4	48·2	48·8	+0·6
Nova band	4641·1	51·2	51·5	+0·3
H_β	4861·5	53·7	53·8	+0·1
N_2	4959·1	54·7	56·1	+1·4
N_1	5006·9	55·3	55·9	+0·6

In a prismatic spectrum the measured width of the bands decrease as we pass from violet to red.

As the beginnings and endings of the bands are often very indefinite, the agreement is perhaps as good as can be expected.

The Radial Velocity of the Broad Band Nebula.

The mean wave-lengths of the beginnings and endings of the bands best suited for measurement, and their velocity displacements, are given in Table III.

TABLE III.

No. of Plates.	Band.	Measured λ's Beginning, Mean, Ending.	Displacements in km./s.	Mean ⊙ Corrn.	Mean Rad. Vel. km./s.	p.e.	Weight.
18	H_δ	4079·3 4101·3 4123·4	−1662 − 48 +1566	−2·3	−51	±3·6	77
16	H_γ	4316·7 4340·0 4363·2	−1650 − 45 +1559	+0·4	−45	±2·8	128
26	H_β	4833·8 4860·7 4887·6	−1711 − 50 +1610	−7·8	−58	±2·4	174
14	N_2	4930·6 4958·6 4887·6	−1726 − 29 (+1668)*	−16·3	−45	±4·3	54
25	N_1	4978·0 5006·1 5034·2	−1732 − 48 +1635	−8·8	−57	±6·3	25
		Weighted mean radial velocity			−51·6	±2·0	

* Assumed equal to mean shift of H_β and N_1 on same 14 plates.

It will be seen that the centres of the bands are very distinctly displaced to the violet by an amount equivalent to $-51{\cdot}6$ km./s. $\pm 2{\cdot}0$ km./s., which is regarded as the radial velocity of the gaseous mass giving the broad bands, *i.e.* the Nova.

Some observers regard the displacements of the fine H, K, and D lines as a measure of the radial velocity of the Nova, a view which is considered by the writer to be erroneous.* There is no definite evidence of oscillations of the centres of the bands due to orbital motion, and within the errors of measurement, the velocity appears to be constant. The diffuse nature of the ends of the bands is held responsible for the rather large discrepancies in the individual measures, but the mean results for the five bands agree satisfactorily.

The measures of the individual plates are given in Tables IV. (H_δ), V. (H_γ), VI. (H_β), VII. (N_2), VIII. (N_1).

TABLE IV.

Hδ. $\lambda = 4102{\cdot}0$. $1\lambda = 73{\cdot}10$ km./s.

Date 1918.	Plate No.	Dark Line.	Bright Band. Begins.	Ends.	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.
June 16	5273	−1718	−1608	+1528	−40	+8·3	−31·7	42·9 A°
17	75	1743	1652	1572	40	7·8	32·2	44·1
18	76	1751	1662	1523	70	7·4	62·6	43·6
21	77	1738	1674	1499	88	6·0	82·0	43·4
22	78	1740	1681	1484	98	5·6	92·4	43·3
28	81	...	1652	1608	22	3·0	19·0	44·6
July 2	87	1674†	1659	1572	44	1·1	42·9	44·2
2	88	1681	1659	1594	33	+1·0	32·0	44·5
5	89	1677	1648	1572	38	−0·3	38·3	44·1
5	90	1664	1649	1623	13	0·4	13·4	44·8
6	91	1681‡	1637	1535	51	0·7	51·7	43·4
8	94	1674	1659	1594	33	1·6	34·6	43·5
10	97	...	1645	1586	30	2·6	32·6	44·2
15	99	1677	1659	1528	66	4·9	70·9	43·6
		1683	1652	1537	58	4·9	62·9	44·7
Aug. 3	5314	...	1703	1586	58	12·9	70·9	45·0
Sept. 12	46	...	1688	1608	40	25·2	65·2	45·1
Oct. 4	61	−1762	−1732	+1630	−51	−27·5	78·5	46·0
	Means	−1705	−1662	+1566	−48	−2·3	−51 ± 3·6	44·2
Equivalent λ's		4078·7	4079·3	4123·4	4101·3	Mean shift −λ0·66.		

* See also Evershed, *The Observatory*, No. 536, p. 86.
† Line duplicated at −1798.
‡ Line duplicated at −1784.

TABLE V.

$H\gamma$. λ4340·6. 1λ = 69·08 km./s.

Date 1918.	Plate No.	Dark line.	Bright Band. Begins.	Ends.	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.
June 18	5276	− 1748	− 1665	+ 1520	− 72	+ 7·4	− 65·0	46·1 A°
21	5277	1727	1658	1506	76	+ 6·0	70·0	45·8
27	5280	...	1628	1529	49	+ 3·2	45·8	45·7
28	5281	1694	1672	1554	59	+ 3·0	56·0	46·7
28	5282	...	1672	1527	73	+ 2·8	70·2	46·3
29	5283	...	1644	1534	55	+ 2·5	52·5	46·0
29	5284	...	1623	1506	59	+ 2·4	56·6	45·3
July 1	5286	...	1658	1568	45	+ 1·4	43·6	46·7
2	5288	1658	1658	1589	35	+ 1·0	34·0	47·0
5	5289	1672	1644	1616	14	− 0·3	14·3	47·2
5	5290	1677	1638	1569	35	− 0·4	32·4	46·4
6	5291	1658	1623	1582	21	− 0·7	21·7	46·4
8	5294	1672	1651	1554	48	− 1·6	49·6	46·4
10	5298	...	1651	1547	52	− 2·8	54·8	46·3
15	5299	− 1672	1670	1619	26	− 4·9	30·9	47·6
Aug. 3	5314	...	− 1643	+ 1627	− 8	− 12·9	− 20·9	47·3
	Means	− 1686	− 1650	+ 1559	− 45·4	+ 0·4	− 45 km./s. ± 2·8	46·5 A°
Equivalent λ's			4316·2	4316·7	4363·2	4339·9	Mean shift − λ0·66.	

TABLE VI.

$H\beta$. λ4861·5. 1λ = 61·68 km./s.

Date.	Plate No.	Bright Band. Begins.	Ends.	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.
June 21	5277	− 1702	+ 1628	− 37	+ 6·0	− 31·0	54·0 A°
22	5278	1659	1573	43	5·6	37·4	52·4
27	5280	1678	1598	40	3·2	36·8	53·1
28	5281	1721	1635	43	3·0	40·0	54·4
28	5282	1733	1647	43	2·8	40·2	54·8
29	5283	1684	1567	58	2·5	55·5	52·7
29	5284	− 1715	+ 1591	− 62	+ 2·4	− 59·6	53·6
Mean 7 plates		− 1699	+ 1606	− 46·6	+ 3·6	− 42·9	53·6
Equivalent λ's		4834·0	4887·5	4860·7			
July 1	5285	− 1715	+ 1622	− 46	+ 1·5	− 44·5	54·1
1	5286	1702	1554	74	1·4	72·6	52·8
2	5287	1690	1591	49	+ 1·1	47·9	53·2
10	5297	1678	1628	25	− 2·6	27·6	53·6
10	5298	1709	1579	65	2·8	67·8	53·3
15	5299	1696	1610	43	4·9	47·9	53·6
22	5301	1770	1647	62	7·8	69·8	55·4
22	5301	1746	1641	52	7·8	59·8	54·9
27	5305	1709	1635	37	10·0	47·0	54·2
27	5306	1721	1610	56	10·2	66·2	54·0
31	5309	− 1702	+ 1647	− 28	− 11·6	− 39·6	54·3
Mean 11 measures							
,, 10 plates		− 1713	+ 1615	− 48·8	− 4·9	− 53·7	53·9
Equivalent λ's		4833·7	4887·7	4860·7			

TABLE VI.—*continued.*

Date.	Plate No.	Bright Band. Begins.	Ends.	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.	
Aug. 3	5313	−1739	+1635	−52	−12·8	−64·8	54·7A°	
3	5314	1672	1579	46	12·9	58·9	52·7	
10	5321	1715	1591	62	15·8	77·8	53·6	
15	5323	1715	1616	49	17·6	66·6	54·0	
22	5329	1727	1604	62	19·9	81·9	54·0	
Sept. 6	5343	1715	1641	37	24·1	61·1	54·4	
12	5346	1727	1591	68	25·2	93·2	53·8	
12	5347	1709	1585	62	25·4	87·4	53·4	
Oct. 4	5361	−1752	+1622	−65	27·5	92·5	54·7	
Mean 9 plates		−1719	+1607	−55·9	20·1	−76·0	53·9	
Equivalent λ's		4833·6	4887·6	4860·6				
Summary.	June	4834·0	4887·5	4860·8	Rad. vel.	−43	53·6	53·8
	July	4833·7	4887·7	4860·7	,,	−54	53·9	
	Aug.–Oct.	4833·6	4887·6	4860·6	,,	−76	53·9	
	Means	4833·8	4887·6	4860·7	Mean	−58 km./s. ± 2·4		

TABLE VII.

N_2. λ4959·1. 1λ = 60·47 km./s.

Date.	Plate No.	Begins.	Ends (assumed.)	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.
July 22	5301	−1621	+1668	+24	−7·8	+16·2	54·4A°
22	5301	1760	,,	−46	7·8	−53·8	56·7
27	5305	1735	,,	34	10·0	44·0	56·2
27	5306	1808	,,	70	10·2	80·2	57·5
31	5309	1669	,,	0	11·6	11·6	55·2
Aug. 3	5313	1723	,,	28	12·8	40·8	56·1
3	5314	1735	,,	34	12·9	46·9	56·3
10	5321	1711	,,	21	15·8	36·8	55·9
15	5323	1711	,,	21	17·6	38·6	55·9
22	5329	1748	,,	40	19·9	59·9	56·5
Sept. 6	5343	1729	,,	30	24·1	54·1	56·2
12	5346	1717	,,	25	25·2	50·2	56·0
12	5347	1754	,,	43	25·4	68·4	56·6
Oct. 4	5361	1748	,,	40	27·5	67·5	56·5
Mean 14 plates		−1726	+1668	−29·1	−16·3	−45 ± 4·3	56·1
Equivalent λ's		4930·6	4986·7	4958·6			

TABLE VIII.

N1. λ5006·9. 1λ = 59·89 km./s.

Date 1918.	Plate No.	Begins.	Ends.	Mean.	☉ Corrn.	Rad. Vel.	Width of Band.
June 27	5280	−1659	+1563	− 48	+ 3·2	− 44·8	53·8 A°
28	5281	1887	1695	− 96	3·0	− 93·0	58·2
28	5282	1545	1635	+ 45	2·8	+ 47·8	53·1
29	5283	1707	1611	− 48	2·5	− 45·5	55·4
29	5284	1773	1575	− 99	2·4	− 96·6	55·9
July 1	5285	1791	1521	−135	1·5	−133·5	55·3
1	5286	1695	1431	−132	1·4	−130·6	52·2
2	5287	1575	1587	+ 6	+ 1·1	+ 7·1	52·8
10	5297	1827	1738	− 44	− 2·6	− 46·6	58·7
10	5298	1743	1599	− 72	2·8	− 74·8	55·8
15	5299	1881	1575	−153	4·9	−157·9	57·7
22	5301	1755	1623	− 66	7·8	− 73·8	56·4
22	5301	1713	1731	+ 9	7·8	+ 1·2	57·5
27	5305	1749	1641	− 54	10·0	− 64·0	56·5
27	5306	1731	1527	−102	10·2	112·2	54·4
31	5309	1761	1749	− 6	11·6	17·6	58·6
Aug. 3	5313	1689	1677	− 6	12·8	18·8	54·7
3	5314	1731	1641	− 45	12·9	57·9	52·7
10	5321	1785	1683	− 51	15·8	66·8	57·9
15	5323	1713	1725	+ 6	17·6	11·6	57·4
22	5329	1695	1677	− 9	19·9	28·9	56·3
Sept. 6	5343	1725	1695	− 15	24·1	39·1	57·1
12	5346	1749	1665	− 42	25·2	67·2	57·0
12	5347	1701	1641	− 30	25·4	55·4	55·8
Oct. 4	5361	1725	1677	− 24	−27·5	−51·5	56·8
Means		1732	1635	− 48·5	− 8·8	−57 ± 6·3	55·9
Equivalent λ's		4978·0	5034·2	5006·1			

As previously noted, the bands H_β, N_2, and N_1 occur near the end of the spectrum, where the focus falls off rapidly. The bands are of such importance that it was nevertheless desirable to measure them, and the results are better than anticipated.

The Narrower Nebular Spectrum.

From August 10th onwards the central portion of the chief nebular bands, N_2 and N_1, about 15 A° wide, is very distinctly stronger than the rest of the bands. Measures indicate that these narrower stronger bands have a different velocity displacement from the broader bands, indicating a receding instead of an approaching mass of gas.

The measures give: Narrow Bands * + 12·5 km./s. ± 4·0 km./s.
Broad Bands − 5·1 km./s. ± 1·8 km./s.
Details of the measure are given in Table IX.

TABLE IX.

Central brighter regions of N_1 *and* N_2 *Bands.*

Date 1918.	Plate No.	Begins.	Ends.	Mean.	⊙ Corrn.	Rad. Vel.	Width of Band.
		N_2. λ4959·1. 1λ = 60·47 km./s.					
Aug. 10	5321	−417	+429	+ 6	−15·8	− 9·8	14·0 A°
22	5329	−387	+417	+ 15	−19·9	− 4·9	13·3
Sept. 6	5343	−399	+520	+ 60	−24·1	+35·9	15·2
12	5346	−387	+484	+ 48	−25·2	+22·8	14·4
12	5347	−417	+478	+ 30	−25·4	+ 4·6	15·2
Oct. 4	5361	−393	+478	+ 42	−27·5	+14·5	14·4
Mean 6 Plates		−400	+468	+ 34	−23·0	+11	14·4
Equivalent λ's		4952·5	4966·8	4959·7	Displacement		+λ0·6
							0·9
		N_1. λ5006·9. 1λ = 59·89 km./s.					
Aug. 10	5321	−443	+443	0	−15·8	−15·8	14·8
22	5329	−389	+503	+ 57	−19·9	+37·1	14·9
Sept. 6	5343	−413	+629	+108	−24·1	+83·9	17·4
12	5346	−389	+485	+ 48	−25·2	+22·8	14·6
12	5347	−449	+479	+ 15	−25·4	−10·4	15·5
Oct. 4	5361	−395	+449	+ 27	−27·5	− 0·5	14·1
Mean 6 Plates		413	+498	+ 43	−23·0	+20	15·2
Equivalent λ's		5000·0	5015·2	5007·6	Displacement		+λ0·7
N_2 and N_1 Mean Radial Velocity				+15	km. per sec.		±5·4
Omitting N_1 5343				+12·5	,,		±4·0

The discrepancies are probably due to the indefinite boundaries of the bands, but the displacement of the narrower and brighter bands towards the red from the centre of the broad N_1 bands is apparent on the enlargements without measurement.

This observation is important as *showing the existence of two separate masses of gas moving away from each other with a velocity of over* 60 *km./s. measured radially.* This recalls Stratton's † observations of the spectra of Nova Geminorum No. 2, in which he found evidence of superposed spectra due to different masses of gas having different composition and different velocities.

He says: "Interpreting the phenomena in terms of the Doppler effect . . . we find survivals of an *a* Cygni spectrum displaced

* Including the most discrepant observation, the figures are +15·0 ±5·4 km./s.

† *M.N.*, 79, 367–368.

as if travelling with Pulse II., whilst the most prominent features of the spectrum are now (March 30th) best summarised as a γ Orionis spectrum displaced with Pulse IV."

It would appear, therefore, that the doubled absorption lines in the spectra of Nova Aquilæ on June 10, 11, and 12 are due to two separate masses of absorbing gas of different velocity, one velocity varying more rapidly than the other.*

The Nebular Band λ4363·4.

It is not until July 10th that this band begins to be prominent. From this date Hγ, which, as we have seen, extends from λ4317 to λ4363, becomes lengthened to λ4387 by the overlapping of the nebular band. At first it is weaker than Hγ, and its beginning is not definite enough for measurement.

On August 3rd the bands are nearly equal in intensity, the central third, due to the overlapping portions of the two bands, is stronger than the outer thirds, due to Hγ alone on the violet and to 4363 on the red side. (See Plate V., 4 and 5.)

By October 4th, 4363 is stronger than Hγ, and when the Nova reappeared in the morning sky the nebular band was much stronger than Hγ, and ultimately replaced it almost entirely.

The measured mean wave-length of the band from 5 plates is 4362·8. Correcting this for the mean velocity found from the hydrogen and chief nebular lines −51 km., and for the earth's motion +20·3 km., we get λ4363·25, as compared with Wright's value from nebulæ λ4363·37 ± ·10.

The separate measures are given below in Table X.

TABLE X.

Nebular Band. λ4363·37 ± ·10 (Wright).

1λ = 68·72 km./s.

Date.	Plate No.	Begins.	Ends.	Mean.	Width of Band. Observed.	Width of Band. Calculated.
Aug. 3	5314	4337·3	4387·4	4362·4	50·1 A°	
10	5321	38·8	87·4	63·1	48·6	
22	5329	39·3	87·2	63·3	47·9	
Sept. 12	5346	38·2	86·9	62·6	48·7	
Oct. 4	5361	38·2	87·0	62·6	48·8	
	Means	4338·4	4387·2	4362·8	48·8	48·2 A°

Corrn. for Radl. Vely. +51 km./s.

,, to Sun −20 ,,

+31 ,, = + 0·45

Corrected λ = 4363·25

* See also Luis Rodés, *Pub. Ast. Soc. Pac.*, **31**, 101–102.

Dr. Nicholson ascribes the lines 4363 and 5007 to the neutral atom of nebulium. The very great variations in their relative intensity in gaseous nebulæ (including Novæ), shown in Table I., vol. 76, of *Harvard Annals*, are not in accord with Dr. Nicholson's statement that "the second line (4363) has always an intensity about $\frac{1}{10}$th of that of the first (5007), a fact which may indicate a common origin."

The λ4640 *Group.*

I. *The* λ4604 *Band.*

Adams and Pease record that, in its later stages, Nova Geminorum No. 2 shows three prominent and well-separated lines at λλ4610, 4641, and 4686.* The measures (Table XI.) show that the two latter are undoubtedly represented by broad overlapping bands in Nova Aquilæ at λλ4641·1 and 4685·8, and it was expected that the more refrangible band overlapping 4641 would be Adams and Pease's first line 4610, but it extends too far to the violet for identity. Where the band has a definite beginning its mean wave-length from nine plates is 4577·8, and assuming the mean of the displacements of the beginnings of the 4641 band (−1703) and the bands N_1, N_2, H_β, H_γ, H_δ, (−1700) we obtain for the mean wave-length of the band λ4603·9, which is too far from 4910 to be accounted for by any error of measurement. The presence of a band at 4610 cannot therefore be determined, and if present it is masked by an overlapping band further to the violet.

TABLE XI.

Measures of Bands 4604, 4641, *and* 4686.

Date 1918.		Plate No.	Band 4604. Begins.	Band 4641 1λ=64·61 km./s. Begins.	Mean.	Ends.	Band 4686 Ends.	Width of 4641 Band.
June	28	5281	4577·9	4614·7	4640·2	4665·6	...	50·9A°
			4577·8	4614·9	4640·2	4665·4	...	50·5
	29	5283	4578·2	...	...	...	(4709·9)	...
	29	5284	...	4614·4	4639·7	4665·0	(4709·5)	50·6
July	6	5291	4577·4	...	...	...	...	...
Aug.	3	5313	4578·0	4615·8	4641·0	4666·1	4711·0	50·3
	3	5314	....	4615·8	4641·0	4666·3	4711·2	50·5
	10	5321	...	4613·8	4640·4	4667·1	...	53·3
	15	5323	4577·9	4614·9	4640·3	4665·8	4711·7	50·9
	22	5329	4577·9	...	...	...	...	...
Sept.	12	5346	4577·6	4615·0	4640·4	4665·8	4711·5	50 8
	12	5347	...	4613·8	4640·9	4668·0	4711·9	54·2
Oct.	4	5361	4577·5	4614·4	4641·1	4667·8	4711·4	53·4
Means			4577·8	4614·7	4640·5	4666·3	4711·4	51·5
Corrn. for Motion ☀ +51, ⊕ −12·9 km./s. =					·6			
					4641·1			

* *Ap. J.*, 40, 295, Plate IX.

Band 4641.—Taking the wave-length of the centre of the band as λ4641·1, the mean measured displacements of the ends of the band are:—

Beginning	$-26\overset{\lambda}{\cdot}4$ =	−1706 km./s.
Ending	+25·2 =	+1628 ,,
Mean		− 39 ,,
Mean ⊙ Corrn.		− 12·9
Radial Velocity		− 51·9 km./s.

Band 4686.—Narrower brighter central portions of band are noted as under:—

Plate	5285	5298	5321	5321	Means
Width in A°	10·2	14·6	11·5	13·6	12·5
Mean λ	4685·3	4686·5	4685·1	4685·8	4685·7

The wave-length 4685·7 corresponds to a radial velocity −26 km./s. (corrected to sun) assuming Fowler's λ4685·98, or to −10 km./s., assuming Wright's λ4685·73. (See also Table IX.)

One of the simplest bright-line star spectra is that of the typical Ob star AGC 8631.

According to Miss Cannon* the spectrum consists of the bright lines H_β, H_γ', H_γ, H_δ', H_δ, H_ϵ', H_ϵ, and in addition three bright lines at λλ4059, 4606, and 4688, the last line being the most prominent line in the spectrum. If we adjust the last two lines to the known wave-length of the enhanced helium line we get 4604 and 4686, the first agreeing with the more refrangible Nova Aquilæ band, whilst the latter is the enhanced helium band.

The bright star bands are much narrower and better suited to accurate measurement in the type star of the next class, Oc, viz. AGC 22763. In this type 4606 is recorded as absent from the Harvard spectra, but a photograph taken here with the 24-inch O.G. prism with 90 minutes exposure on July 16, 1901, shows that it is unmistakably present, but faint.

II. *The* λ4640 *Band.*

The 4640 band is a well-known and prominent band in Novæ generally, at a certain stage, and much has been written about it. Lockyer has collected much information regarding it in his publication "On some of the Phenomena of New Stars." He apparently regards the bright line or band near λ4640 in the stars variously known as Type V, Class O, Wolf–Rayet stars and Planetary Nebulæ, to be a single one common to all these objects and to the Novæ.†

A careful study of the broad band in Nova Aquilæ has led me to conclude that the band is complex and that at least three lines

* *Harvard Annals*, 28, 233, Table XXII.

† See "On some of the Phenomena of New Stars," Table VI., p. 26.

are involved, the strongest line peculiar to Novæ superposed on the pair found in Wolf-Rayet stars by Campbell at λλ4636 and 4651.

In Nova Aquilæ the strongest component of the group of overlapping bands, extending from λ4578 to λ4712, is the well-marked one extending from λ4615 to λ4667. (See Plate III., 2, 3, and 6.)

Table XI. shows the measures of this central strong band and the beginning and ending of the entire group.

Correcting the measured mean position of the central strong band 4640·52 for the previously obtained radial velocity and for the earth's motion −51 and +12·9 km./s., we obtain 4641·1 as the wave-length of the central band agreeing with Adams and Pease (*ibid.*). This wave-length gives not only a radial velocity agreeing with that previously found (as it should), but also displacements of the beginning and ending of the band in accordance with those found for other bands, viz. corrected to sun −1716 and +1614, as compared with −1707 and +1603 for the mean of the bands H_β, H_γ, H , N_2, and N_1. (See Table III.)

III. *The* λ4634 *and* λ4650 *Bands.*

The Nova band 4641 is distinctly winged on the violet side, particularly on Plates 5314 and 5346, August 3 and September 12. (See Plate III., 3 and 6.)

The measures give 4608·6, 4607·9—*i.e.* the wing extends to the violet 7·2 and 7·1 respectively, giving λ4634 for the centre of an overlapping band.

On other plates the 4641 band is distinctly winged on the red side. This is particularly noticeable on Plate 5284, June 29, the wing extending to 4673·7—*i.e.* λ8·7 beyond the 4641 band, an indication of a band at 4650, possibly identical with Campbell's band at 4651 in Wolf-Rayet stars. (See Plate III., 2.)

IV. *The* λ4686 *Band.*

Assuming the least refrangible band to be due to the enhanced helium line λ4685·98 (Fowler),* the mean displacement of the last six accordant observations (Table XI.) of its ending is +1630 km./s. The agreement with the displacement of the ending of the band 4641 for the same six plates, viz. +1649, corroborates the wave-length.

Alternatively, correcting the observed wave-length of the ending of the band for the measured displacement of the 4641 band, viz. +1649, we obtain λ4685·68.

Using the mean displacement of H_β and N_1 on the same six plates, viz. +1638, we obtain 4685·85, which is exactly the mean of the wave-lengths obtained by Fowler in the laboratory (λ4685·98), and Wright from nebulæ (λ4685·73 ± ·10).

* *M.N.*, 73, 66.

It would appear, therefore, that the very broad band extending from 4578 to 4666, 88 A° wide, is a complicated one, consisting of at least five overlapping bands, the centres of which are at λλ4604, 4634, 4641, 4650, and 4686, the most prominent being 4641 and 4686.

Table XII. compares the Nova bands as thus interpreted with the bands found by Campbell and Miss Cannon in Wolf-Rayet stars.

TABLE XII.

Lunt.	Cannon.			Campbell.			
Nova Aquilæ.		Int. in Type Stars.		λ in Type Stars.		Mean λ in Wolf-Rayet Stars.	
λ.	λ.	Ob.	Oc.	Ob.	Oc.	vis.	photo.
4604	4606	12	0	460–			
4634	4633	0	20	4628	4638 {v / p	4633_7	4636_8
4641							
4650						4652_{13}	4651_{12}
4686	4688	100	40	4688	{4687 v / 4690 p	4688_{30}	4688_{23}

The suffixes indicate the number of stars included.
v = visual. p = photographic.

It would appear that the Nova band at 4650 is the same as Campbell's Wolf-Rayet band at 4651, and possibly the dark line at 4649·2 so strong in the B type star ε Orionis * (Harvard) is identical with it.

Campbell states that "the photographs show, even better than the visual observations, that the blue bands at 4688 and 4652 are often associated in the same spectrum. The band at 4636 is distinctly separate from those at 4652 and 4626 and is found in several stars" (Wolf-Rayet stars). If we adjust these wave-lengths and the Harvard 4606 in Ob stars to the now known wave-length of the helium line 4686, the agreement with the Nova bands is remarkable, and their wave-lengths agree with those found for the bands in Nova Geminorum No. 2 by Adams and Kohlschutter, and in Nova Lacertæ by Wright, as shown below.

Wolf-Rayet Stars.	Nova Aquilæ.	Nova Gem. (A. & K.)	Nova Lacertæ (Wright) Maxima. †
4604 (Ob)	4604	4605	
4634	4634	4632	4633
	4641	4641	4641
4650	4650	4649	4649
4686	4686	4687	

* Keeler finds a very strong line at λ4650 in ε and ζ Orionis and the trapesium star band 628. *A. and Ap.* xiii. 491.

† *L.O.B.*, 6, 98.

It would appear, therefore, that 4641 is a separate band characteristic of Novæ, superposed on the Wolf-Rayet bands 4634 and 4650, which in Campbell's list are remarkable in never appearing together in the same star.

There is, however, considerable uncertainty in the wave-lengths of the star bands, and perhaps too much stress should not be laid upon the above agreement.

Origin of the Bands of the λ4640 *Group.*

Only one of the bands can be definitely assigned to its origin, viz. the helium band 4686.

Lockyer, Baxendall, and Butler * suggest that the 4650 line in Wolf-Rayet stars may be due to the carbon double 4647·6 and 4650·8, and Adams and Kohlschütter † have called attention to the close agreement between the bands in Nova Geminorum and nitrogen lines, and they quote Wright's ‡ previous observations of Nova Lacertæ.

Further work on the enhanced nitrogen lines in this region is desirable, to trace possible variations in intensity under varying conditions.

In view of the further evidence shown by other bands in Nova Aquilæ, presently to be considered, it seems probable that nitrogen bands are involved in the 4640 group, but it is not certain that they are alone responsible for the bands in Novæ and Wolf-Rayet stars.

The Nitrogen Bands: The λ4097 *Band.*

On June 27, 28, and 29 the bright H_δ band is very definitely extended towards the violet, and it seemed probable that this extension was caused by an overlapping band due to the enhanced nitrogen line 4097·45 first recorded as abnormal by Lockyer, Baxendall, and Butler. §

This is a line first ascribed to silicon (Group IV.) by Lockyer, but which I concluded could not be due to silicon and pointed out that it was an important stellar line rivalling H_δ in intensity in 29 Canis Majoris.‖ The above observers afterwards proved that the line is due to nitrogen under high temperature conditions, and stellar spectra indicate that this line may stand alone as the evidence of nitrogen at high temperatures.

Taking the wave-length of the line as 4097·45,¶ the displacement of the beginning of the band on plate 5283, June 29, was − 1643 km./s., which is in excellent agreement with the displacement of the beginning of the H_γ band on the same date, viz. − 1644 km./s., and the mean H_γ − 1650.

The best evidence of the duplicity of the H_δ band is shown on

* *Proc. Roy. Soc.*, 82, 543–544.
‡ *L.O.B.*, 6, 98.
‖ *Ibid.*, p. 538.
† *Ap. J.*, 36, 316–317.
§ *Proc. Roy. Soc.*, 82, 532.
¶ *Ibid.*, p. 537.

plate 5346 of September 12, when the hydrogen component had faded sufficiently to show the overlapping portions of the bands very definitely stronger than the wings at each end, due to nitrogen 4097 and H_δ respectively, an appearance not shown by the beginning of H_γ and the ending of the nebular band 4363 which are very definite. (See Plate VI., No. 5, and V., No. 6.) As a mean of four settings the following wave-lengths were obtained :—

			Means.	Corrd. λ o·36.
	4075·0			
Nitrogen.	4078·9		4097·05	4097·4
4097	4119·1	Hδ	4101·55	4101·9
	4124·2			

which, corrected for motion of Nova − 51 km. and earth + 26 km., give 4097·4 and 4101·9.

The λ3995 *Band.*

In laboratory spectra obtained from vacuum tubes under high temperature conditions the line 4097·45 is accompanied by an equally strong line at 3995·1,* the strongest nitrogen line given by Exner and Haschek.

In stars, however, the evidence points to the relative weakening of 3995·1 at higher temperatures, as the following intensities show :—

†	29 Canis Majoris.	τ Canis Majoris.	ε Orionis.	β Centauri.	γ Orionis.
3994·9	0	1	2	3	5
4096·9	18	6	4	2	1

These two lines therefore appear to afford a valuable indication of relative temperature.

Nova Aquilæ in its early stages is undoubtedly at an excessively high temperature as shown by the titanium spectrum, consisting entirely of enhanced lines, and it should occasion no surprise if 3995·1 were absent from Nova Aquilæ. Supposing it to be present with the same displacement as the nitrogen line 4097·45 and H_γ, viz. 1643 km./s., it should commence at λ3973·2.

Close to this place the broad bright H_ϵ becomes intensified, as though by an overlapping band, particularly so on plate 5282 taken June 28 where the band starts at λ3972·7. (See Plate II. No. 2.)

Not only so, but assuming the red end of the band to have the same displacement as H_γ on June 29 (5283), viz. + 1534, it should end at λ4015·5.

The end of a bright patch on the same plate occurs at 4015·3, a little beyond the iron line 4014·7. (See Plate.)

* *Proc. Roy. Soc.*, 82, 535.

† *Harvard Annals*, 28, 233 and 236, Tables XXII. and XXIII.

Almost central on the band is a heavy, broad absorption band, 13 A° broad, with its centre at 3997·6. At later stages this dark band is not present. Adams and Kohlschütter record a bright band in the spectrum of Nova Geminorum No. 2 with boundaries 3986·0 and 4007·6, mean 3996·8. This is probably the same band.

The λ4103 *Band.*

The nitrogen line 4103·46, which accompanies 4097 in laboratory spectra, is so close to H_δ that it is difficult to say whether H_δ is abnormally extended towards the red, as the ends of the bands are rather indefinite, but so far as the measures go they indicate that the band is so extended, as the mean ending for June 16–22 is at λ4122·8, and June 28–29 at λ4124·5, a difference of λ1·7, the difference between H_δ and the nitrogen line being λ1·5.

The appearance of the 3995 band attributed to nitrogen was short-lived, and only recognisable with any degree of certainty on June 27, 28, 29. On July 10, H_ϵ shows evidence of being overlapped by the 3995 band, but the beginning of the 4097 band, which would disappear under cooler conditions, is no longer differentiated from the continuous spectrum. The latter band is, however, present in August and September.

The λ4447 *Band.*

Exner and Haschek record a nitrogen line at 4447·23, Int. 20. With the same displacements as before, this should give rise to a band from 4422·8 to 4469·9. On June 29, Plate 5283, two bright patches were measured with a broad dark absorption patch superposed. The bright patches begin and end at 4424·3 and 4469·5 respectively.

The λ4631 *Band.*

The nitrogen line 4630·73, Int. 15, falls on the 4640 group, one component of which is at λ4634 and cannot be differentiated.

The λ4380 *Band.*

A faint line given by Exner and Haschek, Int. 1, at λ4379·8 is another line of abnormal behaviour, and under certain conditions it was found by Lockyer, Baxendall, and Butler to be the strongest line of the spectrum. If present, it should, with the previously used displacement, give a band 4355·8–4402·2.

On June 29, Plate 5283, H_γ ends in an unusual bright maximum (see Plate V., No. 2) beginning at 4354·8, and another bright patch ends at 4401·5, whilst there is a stronger narrow brighter patch, 8 A° wide and with well-defined edges at 4382·2, followed by a dark region, and then by the bright patch ending at 4401·5.

The weakness of the portion to the red side of H_γ makes the nitrogen origin of the strong red end of H_γ doubtful.

Further confirmation of the presence of bright nitrogen bands is afforded by the record of a band at 5645–5706 * on July 1 by Paddock. Exner and Haschek record a strong nitrogen line at 5679·70, Int. 20. With the above displacements, −1643 and +1534, this would give a band from 5648·58 to 5708·75.

Later measures by the same observer are closer to the calculated wave-lengths, viz. 5647–5708, with a note "strong after June 25."† Moreover, Paddock records a band at 5665 which may be Exner and Haschek's nitrogen line 5666·78, Int. 10, and also a band at 5944 which may be Exner and Haschek's nitrogen line 5941·9, Int. 10.

Wright, ‡ discussing the spectrum of Nova Lacertæ, says: "Aside from the apparent absence of this strong line (3995) from the spectrum of the star, correspondence between the two spectra (Nova and nitrogen) is striking. . . . As the matter stands, however, the existence of nitrogen in the star can hardly be said to be proved."

Adams and Kohlschütter§ conclude that "in view of the evidence, we are inclined to consider the presence of nitrogen in Nova Geminorum as probable, although we would agree with Mr. Wright in considering it as hardly proven."

In view of the cumulative effect of so many indications, and the further evidence afforded by Nova Aquilæ, the presence of nitrogen bands in Novæ seems to me to be established.

Taking all Exner and Haschek's lines of intensity 10 and brighter, and including the lines of normal behaviour discussed by Lockyer, Baxendall, and Butler, 4097, 4103, and 4380, and computing the beginnings and endings of corresponding bands from the displacements obtained for H_γ and H_β on the same plates, we get the comparison shown in Table XIII., supplemented by Paddock's observations of the less refrangible bands outside the region photographed here, and by Adams and Kohlschütter's values for Nova Geminorum.

The measures of the first and third bands (Table XIII.) refer to the means of three plates, 5281, $5282\frac{3}{4}$, 5283 (June 28–29), and for the calculated values, the mean displacements of H_γ and H_β for the same plates have been used.

The second band is calculated from the figures for H_δ on Sept. 12 given in Table IV. to compare with the measured values for the nitrogen band on the same date. The fourth and fifth bands were measured on the last plate only, and for the calculated values the mean displacement of H_γ and H_β for the same plate has been used.

The mean displacement of H_γ and H_β for August 3 have been used for the calculated values of Paddock's bands observed on that date.

* *Pubns. Astr. Soc. Pac.*, 30, 248.
† *Ibid.*, 31, 51.
‡ *L.O.B.*, 6, 98.
§ *Ap. J.*, 36, 316.

TABLE XIII.

Nitrogen Bands.

Nitrogen E. & H. λ	Int.	Bands Calculated.	Bands Observed in Nova Aquilæ.	Centres. (Corrected for Motion.)	Nova Geminorum. A & K. Centres.	Remarks.
3995·26	50	3972·8 4016·3	3972·4 4015·7	3994·8	3996·8	3986·0 4007·6
4097·43	3*	4074·4 4119·4	4074·9 4119·1	4097·4		
4103·46	3*	4080·4 4125·0	... 4124·3 †			
4379·75	1*	4355·4 4402·4	4354·8 4401·5	4378·9	4379·0	bright max
4447·23	20	4422·6 4470·2	4424·3 4469·5	4447·7		
4630·73	15	In 4641 Group.		Centres Paddock.‡		
5666·78	10	5634·9 5697·3	5634 5695	5665	5665·6§	bright bd.
5679·70	20	5647·8 5710·3	5647 5708	5678	5680·2	bright bd.
5941·9	10	5908·5 5973·9	5912 5976	5944	5942·3	f bright bd.

The remaining features of the 1918 spectra and a discussion of the measures of the 1919 spectra are reserved for a further communication.

Royal Observatory,
Cape of Good Hope,
1919 *October* 31.

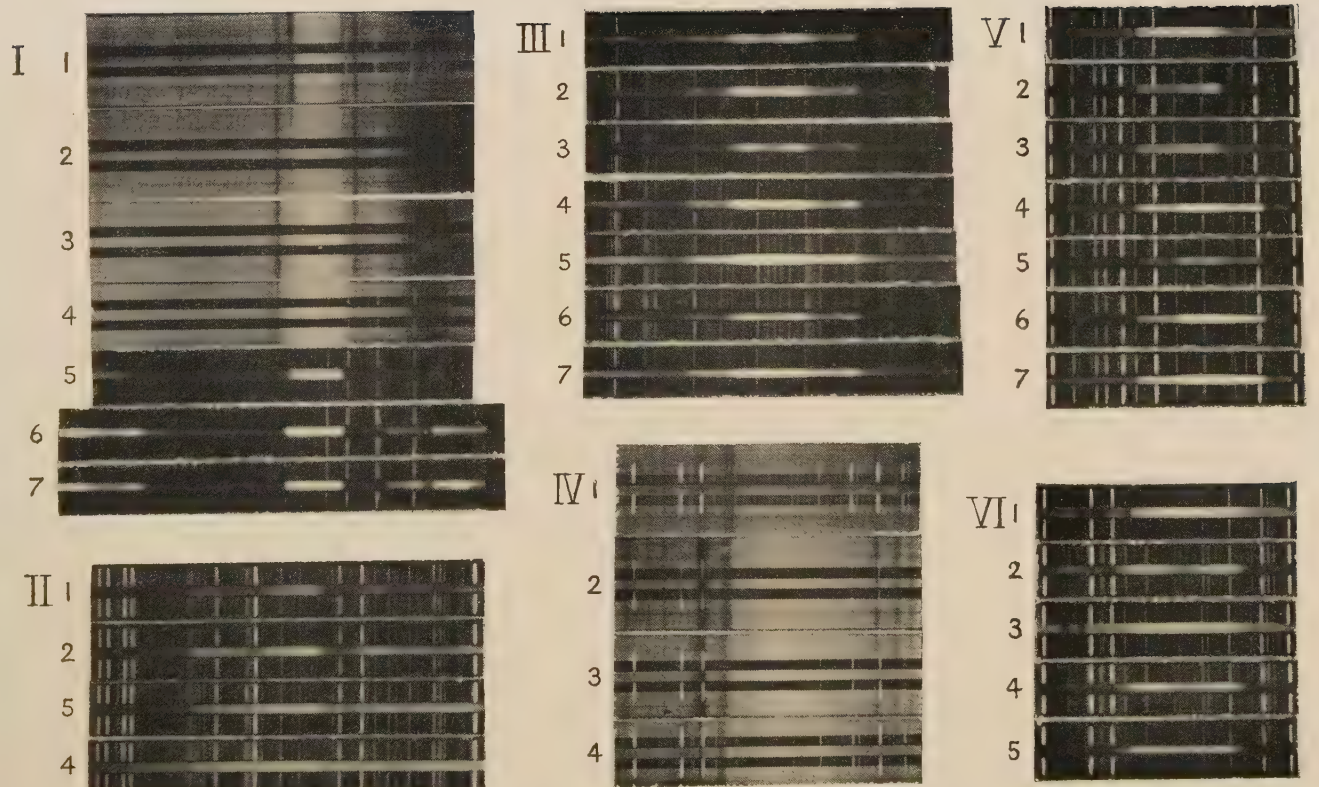
* Lockyer's lines of abnormal behaviour.

† Perhaps indistinguishable from Hδ. A. & K. = Adams and Kohlschütter, *Ap. J.*, **36**, 306.

‡ *Pubns. Ast. Soc. Pac.*, 30, 248.

§ Calculated from violet edge of bright band at 5649·2, assuming width proportional to H 28·2 A°.

Note added 1919 *November* 5.—The important paper by Evershed (*M.N.*, **79**, 468–490) on the spectrum of Nova Aquilæ has only to-day come into my hands as a reprint, kindly sent by the author, which explains the absence of any reference to it in my paper despatched two days ago.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KEY TO PLATE.

Group I.

Region, H_β, N_2, N_1.

Plate.			
1	5260	June	10
2	5266		11
3	5269		12
4	5272		15
5	5278		22
6 } *	5313	Aug.	3
7 }	5346	Sept.	12

The strong dark $H\beta^1$ fades and becomes bright in 4 when $H\beta^2$ becomes narrow and $H\beta^3$ appears. 5 shows $H\beta$ bright and alone. 6 and 7 show N_2 and N_1. The last shows the central brighter narrower bands N_2 and N_1.

Group II.

H_ϵ Band.

1	5281	June	28
2	5282		28
3	5287	July	2
4	5289		5

No. 4 shows H_ϵ terminating in a dark line. 1 and 2 show the stronger region attributed to Nitrogen 3995.

Group III.

Bands of 4641 Region.

1	5280	June	27
2	5284		29
3	5314	Aug.	3
4	5323		15
5	5329		22
6	5347	Sept.	12
7	5361	Oct.	4

All show the abrupt ending of 4686. 4, 5, and 7 best show the beginning of 4604. 2 and 3 best show the wings due to 4634 and 4650. 4641 is best seen as the most prominent band in 3 and 6.

Group IV.

H_δ Band.

1	5260	June	10
2	5266		11
3	5269		12
4†	5272		15

The progressive shift of $H\delta^1$ towards the violet is well seen. The iron lines are in alignment. 4 shows $H\delta^2$ much narrowed. $H\delta^3$ is not seen.

Group V.

$H\gamma$ and Nebular Band 4363.

1	5277	June	21
2	5284		29
3	5291	July	6
4	5313	Aug.	3
5	5314		3
6	5346	Sept.	12
7	5361	Oct.	4

No. 1 shows bright $H\gamma$ with dark $H\gamma$. 2 shows stronger red end of $H\gamma$. 4 shows nebular band 4363 superposed on $H\gamma$. 5 shows overlap strengthened. 6 and 7 show 4363 stronger than $H\gamma$.

Group VI.

$H\delta$ Band and Nitrogen 4097.

1	5275	June	17
2	5280		27
3	5299	July	15
4	5323	Aug.	15
5	5346	Sept.	12

Nos. 1 and 3 show dark $H\delta^1$. 2, 4, and 5 show extension of $H\delta$ to violet due to 4097. 4 and 5 show strengthened overlap of the two bands. 3 shows the very strong dark bands at 4054, 4060.

* The enlargement shows, on the left, half the 4641 band and the 4686 band. Compare Group III., 3 and 6.
† Compare with Group VI., No. 1, two days later.

The Spectrum of Nova Geminorum II., 1912. (*Summary.*)

By F. J. M. Stratton, D.S.O., M.A.

(*Communicated by the Director of the Solar Physics Observatory, Cambridge University.*)

This paper is a brief summary of the complete investigation of the series of plates secured between 1912 March 15 and 1913 March 12 at Cambridge and at Allegheny. The full paper is being published in the *Annals* of the Solar Physics Observatory. The material includes plates taken on 40 out of the first 46 days after the discovery of the nova. In addition, all published investigations of the nova's spectrum have been examined, and a detailed account of the changes occurring in the spectrum day by day has been given, all available sources being compared and utilised in this work.

Here it is only possible briefly to summarise the changes which can be traced. The first spectrum secured at Michigan, 1912 March 13, the day after discovery, showed an absorption spectrum which can be identified as consisting of hydrogen, enhanced calcium, iron, titanium, and silicon—the lines being well displaced to the violet from their normal positions. The type has been described as A 5. A few wide emission lines were noted on the red side of the hydrogen and calcium absorptions. The absorption lines grew rapidly stronger and more numerous, and at the same time the bright bands stood out more from the continuous spectrum. The absorption spectrum became intermediate in type between α Cygni and γ Cygni, and the multiple of the wave-length by which it was displaced to the violet after a slight increase began steadily to decrease. At the same time a second set of α Cygni absorption lines appeared (March 16) more displaced to the violet than the first, and the bright bands developed a complex structure with several maxima. The absorptions moved in slowly towards their normal undisplaced positions, the first one weakening and being lost in the bright bands. On March 19 a third absorption displaced still more to the violet appeared. This was found at first for hydrogen and helium alone, but nitrogen and oxygen were soon found to be represented by lines with nearly equal displacements, and by March 25 we had present in the star two well-defined absorption spectra of different types. The A-type spectrum, containing hydrogen, iron, titanium, helium, and calcium, was displaced by an amount which, interpreted as a Doppler effect, would mean a velocity of approach for the absorbing gases of just over 800 km./sec.; the B-type absorption spectrum, consisting of hydrogen, helium, oxygen, carbon, nitrogen, showed a velocity of approach of about 1450 km./sec. The bright bands, which at this stage mostly represented the A type of spectrum, settled down to a structure which, for hydrogen at least, remained normal for some weeks. There was a bright central portion with two maxima displaced by equal amounts (about 6 tm.)

from the centre of the band. The width of the central bright portion varied directly as the wave-length, being about 24 tm. at Hγ. The bands were flanked by much weaker wings, and if the wider band included the second absorption and stretched out to the third absorption, it was just double the width of the central band, the two bands having a common centre. The centres of the bands were displaced about 2 tm. to the red.

The next feature to be noted (March 26) is the coming up of the broad bright band at 464 $\mu\mu$. On March 30 the nova showed its most complex spectrum. The B-type absorption spectrum had become much stronger than the A type, the strongest lines being reminiscent of the spectrum of γ Orionis. The two hydrogen absorption lines were both doubled, the bright centres of the two pairs corresponding in displacement to the respective absorption lines of the A or the B type of the rest of the spectrum. From this date on we lose the steady progressive sequence of spectra, the star passing through a curious oscillating stage both of magnitude and of spectral type. A period of just over three days is suggested, accompanied by progressive change. As the star gets fainter the absorption spectrum is replaced by a bright-band spectrum—the decrease in intensity of the continuous spectrum causing the apparent loss of light. The γ Orionis spectrum provides most of the strong absorption lines at maximum brightness and most of the bright bands at minimum.

As the iron and titanium drop out of the spectrum bright bands corresponding to the principal nebular lines gradually grow stronger, the first traces having been found on March 28 at Heidelberg. By April 23 a bright-band spectrum due mainly to hydrogen, helium, and nebulium, had been formed ; the spectrum changed during the summer to a more purely nebular type. The bright bands had become by 1912 December chiefly nebular, but the presence of such lines as He λ4686 indicated a transition to the Wolf-Rayet stage. The double maxima in the bright bands persisted from the earlier stages. Although these maxima varied in relative brightness from time to time, and although they belonged for the most part to different bands from those of 1912 March and April, yet they were displaced from the centres of the bands and from the undisplaced origins by the same multiple of the wave-length. This points to a striking persistence in the causes underlying the complexity of the bright bands, and a similarity in the effects of these causes on the spectral lines of different elements.

Before discussing possible explanations, two more facts must be briefly passed in review. From the earliest date there were present narrow dark lines in the undisplaced positions of the calcium lines H, K. Compared with the absorption lines previously discussed, these lines were narrow and stable in position. There is slight evidence for a very small movement towards the violet in the latest plates on which the lines were visible (the latter half of April), but the evidence is not convincing owing to possible blends

with displaced lines of different origins. Accompanying these two lines there are a certain number of much weaker narrow dark lines whose identity has not been previously established in novæ. They are very weak for the most part and difficult to measure, but they clearly indicate in this nova a weak undisplaced α Cygni spectrum, possibly due to a survival in a quiescent state of the gases which gave the first set of absorptions.

The above is a summary of the chief facts about Nova Geminorum II. which the examination of the spectra has revealed. Most of the facts mentioned above are typical of bright novæ in general. Thus the replacement of a displaced A-type absorption spectrum by a B-type absorption spectrum is found in Nova Aquilæ III., 1918. The passage of the complex bright-band spectrum with little change of band structure from an A type through a B type to nebular and Wolf-Rayet types has been found in several novæ, as also a tendency for successive like features in the bands to be separated by constant differences of frequency. The numerical relationships, however, between the widths of elements of the bands have not been found to be quite so simple in other novæ. The sharp H and K lines (and also D_1 D_2 of sodium) have been found in a succession of novæ and a weak undisplaced spectrum in the early plates of Nova Persei. Further, it may be noted that to most of the transitory types found in the early stages of the nova there correspond stellar types of a considerable degree of permanence or stability.

Explanations of the main phenomena in terms of electric or magnetic fields have failed, and the evidence is decisive for the interpretation of the observed results in terms of the Doppler effect. The displacements vary directly as the wave-lengths, and are apparently independent of the element concerned or of any particular grouping of lines in the spectrum of an element. Most of the phenomena can be explained in terms of streams of gas issuing from a number of points in one body, points which share in the rotation of that body. The changes of displacement of the absorptions in the early stages, the oscillations between an absorption and a radiation spectrum, and the ultimate replacement of an absorption spectrum by a bright-band spectrum, can be so explained. The latter change takes place as the main external radiation passes from the disturbed centre to the outlying and outflowing envelope. The expansion of this envelope has been recently indicated both for Nova Persei (1901) and for Nova Aquilæ (1918). There is no evidence for the formation or presence of a few nuclear bodies in these envelopes, though there is evidence in Nova Aquilæ for a varying distribution of velocity over the disc into which the star is seen to expand. This would accord with the hypothesis of expanding streams or jets.

The progressive change of elements represented in the spectrum towards the lighter gases may be due to disintegration, or to the effect of changing environment, in particular of growing tenuity, upon the spectrum of simple elemental gases. The time is hardly

ripe yet for a final theory of the evolution of a nova, still less of its outburst, but knowledge is growing very rapidly; much may be hoped from the study of the complex phenomena of Nova Aquilæ III., 1918; the simpler case of Nova Geminorum II. has already helped to explain some details of the spectrum of the later star, and it is hoped that this investigation may help in other important respects.

The Orbits of 20 *Double Stars.* By J. Jackson, M.A., B.Sc.

(*Communicated by the Astronomer Royal.*)

At Greenwich we are engaged in compiling and discussing all the double-star observations made with the 28-inch refractor since 1894. The observing list is a miscellaneous one, and includes most pairs, showing orbital motion, within the limits of declination for which the telescope can be used. In discussing the observations new orbits have been computed where such seemed advisable. No orbits have been computed for pairs for which an orbit has already been computed from observations extending over a whole revolution or nearly a whole revolution. The work has therefore been confined to pairs for which the observations extend over about half the period of revolution. There are several orbits for which the greater part of the 360° of position-angle has been observed, but for which observations extend over only a comparatively small part of the period, *e.g.* Σ1639 and Σ2525. On the other hand, there are several for which the observed arc is small, but the fraction of the period covered by the observations large, *e.g.* Σ73.

Various methods have been used in determining the orbits. In all cases the position-angles, θ, have been plotted against the time, t, and a smooth curve drawn. The same has been done with the distances, ρ, in those cases where the variation in the measured distances seemed to be significant. The polar co-ordinates, ρ, θ, were then plotted and a smooth curve drawn through them. The constancy of the rate of description of area was then tested, at first by the expression $\rho_1\rho_2(\theta_2 - \theta_1)$, and later by the use of a planimeter. The (θ, t) and (ρ, t) curves were then adjusted in the direction indicated by the law of areas with as little violation of the observations as possible. In this way values of ρ, θ, satisfying the law of areas, were obtained for various epochs. In the earlier computed orbits Kowalsky's method was generally followed. In this method the apparent ellipse is obtained analytically from five points. When this method was followed the five points used are given. The apparent ellipse then gave the elements which determine the size, shape, and position of the orbit. The semi-major axis has frequently been altered to get a better fit with the measured distances, so that the initially assumed distances do not agree with the final value of the semi-major axis. With the elements thus obtained the position-

angles at various well-determined epochs were converted into mean anomaly and by a least-square solution the time of periastron and the mean motion were computed.

In many of the closer pairs Zwiers' method was followed. For this method the observed arc was drawn as indicated above, and then the complete apparent ellipse was drawn. The numerical work is shorter than for Kowalsky's method, but the apparent ellipse must be carefully drawn and measured. Zwiers' method is not capable of giving such accurate results as Kowalsky's, but in most cases it can be easily made as accurate as the observations justify. In the drawing of the ellipse, however, there is a tendency to draw the ellipse of minimum period, but when the observations cover a sufficient arc there should not be such a difficulty.

For Σ1639 Thiele's method has been used. This method depends on the use of three complete positions, and is very useful for very eccentric orbits. The three positions used were, of course, obtained by a graphical adjustment of all the observations available.

The following notation has been used :—

T = time of periastron.
P = period in years.
$n = 360°/P$ = the mean annual motion. This has been treated as positive when the motion is in the direction of increasing position-angles, negative in the opposite direction.
a = semi-major axis of the true orbit, expressed in seconds of arc.
e = eccentricity.
i = inclination of the orbit plane to the tangent plane of the sphere. It is, of course, impossible to distinguish between positive and negative values of i.
Ω = position-angle of the line of intersection of the orbit plane and tangent plane. Ω is always taken as less than 180°.
ω = angle between the line of nodes and the periastron, measured from the line of nodes in the direction which projects to increasing position-angles.

There is generally no ambiguity in these terms except ω. Professors Campbell, Aitken, and Hussey have suggested that ω be measured in the direction of motion in the orbit plane (see Aitken, *Binary Stars*, p. 72). This means that for stars in which the direction of motion is that of decreasing position-angles, ω is to be changed to $360° - \omega$. This suggestion has the advantage of giving ω a unique determination for spectroscopic binaries which should otherwise be affected with the sign $\pm$. For visual double stars the determination of ω is unique. In support of the method here adopted, it may be stated that there is no apparent advantage to be gained by the adoption of a notation designed to hide the indeterminate elements in spectroscopic systems. The definition of ω I have adopted has the advantage of conforming to usual practice

in the past. Elements are often published with no definite statement as to their interpretation, and in view of the new suggestion the greatest care will have to be exercised in the use of published elements. For example, at the end of Aitken's book there are collected the elements of 87 double-star orbits: my list contains 8 of these with decreasing angles, viz. Σ1639, Σ1768, Σ1879, OΣ285, Σ1938, Δ15, Σ2525, OΣ387. A comparison with the elements I have obtained indicates that all except the last two, by Doberck, follow the notation I use. That is, the orbits by Lewis, v. Biesbroeck, Lohse, and Evans follow it. On consulting the original work of Doberck, I find that one orbit is given in each notation and that Aitken has properly put both into his notation. It should be added that See in *Evolution of the Stellar Systems* follows Aitken's notation.

To compute positions from the elements I give: Take the true anomaly increasing or decreasing with the time according as n is positive or negative, add ω, project into the same quadrant in the tangent plane, and add Ω.

A short ephemeris is given for each binary. The dates given, unless otherwise stated, refer to the beginning of the year.

The hypothetical parallaxes have been computed from the formula $\pi = 2^{-\frac{1}{3}} a \cdot P^{-\frac{2}{3}}$, which assumes the mass of the system to be twice that of the sun.

The following elements were obtained for β1111 from observations extending from 1889 to 1915:—

T	1919·94	a	0″·240	i	43°·5
P	46·82 years	e	0·150	Ω	50·6
n	7°·69			ω	127·4

But these have already been superseded by elements given by Aitken (*Publ. A.S.P.*, 31, 285), who employed observations to 1919.

In the heading for each star are given the number in Burnham's General Catalogue, the common name of the binary, and the B.D. number. The right ascension and declination are for 1925·0. The magnitudes are taken from Burnham's Catalogue and the spectral type from the Harvard publications. The source of information for the proper motion and parallax is given.

It is intended to give fuller details, and a comparison between the observed and computed places, in a Greenwich volume.

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482. Σ73. B.D. 22°, 146.

R.A. $0^h\ 50^m\ 56^s$. Mags. 6·2, 6·8. P.M. ″·142 in 103°·4 (Boss).
Dec. +23° 14′. Spec. Ko.

Several orbits of this pair have previously been computed. The periods range from 109 to 349 years. The period given

below, 124·2 years, may be in error by 10 years or more. The eccentricity is large and rather uncertain; if the period is longer than that given the eccentricity will be less. The observations used extend from those by Struve in 1832 to those at Greenwich in 1913. The actual data from which the apparent orbit was computed was

1835	312°·9	0″·97
1855	336·5	1·25
1875	354·5	1·30
1895	13·5	1·20
1915	39·1	0·96

the last being a slight extrapolation. The arc described in 80 years was thus less than 90°, but the remaining 270° will be described rapidly. All the observations are well represented by the orbit given below. By 1940 this will be a close pair.

T	1815·9	*a*	0″·97	*i*	41°·2
P	124·2 years	*e*	0·708	Ω	105°·1
n	+2°·897			ω	76°·5

1915	38°·8	0″·949
1920	47·6	0·863
1925	58·6	0·763
1930	73·4	0·636
1935	97·7	0·459

The hypothetical parallax is 0″·0309.

1144. Σ228. B.D. 46°, 536.

R.A. $2^h\ 9^m\ 13^s$. Mags. 6·7, 7·6. P.M. ″·112 in 225°·3 (Greenwich).
Dec. +47° 8′. Spec. Fo. π+″·069 ±″·009 (Sproul).

The periods of previously computed orbits range from 88 to 205 years. For the orbit given below the observations extend from Struve in 1831 to Aitken in 1912. The apparent orbit was deduced from the following positions:—

1830	262°·3	1″·08
1860	286·6	0·96
1880	327·0	0·45
1890	39·0	0·36
1910	101·7	0·76

The angular motion has thus been about 200°, but as the maximum separation about the present time is not completely covered by

observations, the period must be uncertain by perhaps 30 years. The maximum separation about 1840 seems to be just covered by observations. If the period has to be increased so also will the eccentricity.

T	1894·5	a	0″·974	i	61°·3
P	167·4 years	e	0·313	Ω	99·7
n	+2°·150			ω	303·7

1915	109°·9	0″·781
1920	117·6	0·768
1925	125·9	0·732
1930	135·1	0·685
1935	145·9	0·635

The hypothetical parallax is ″·0255.

1761. Σ412. B.D. 23°, 473.

R.A. $3^h\ 29^m\ 59^s$. Mags. 6·6, 6·7. P.M. ″·032 in 146°·6 (Boss).
Dec. +24° 13′. Spec. A2.

The following orbit has been deduced by Zwiers' method from observations beginning with Struve in 1828 and ending with the Greenwich observations of 1913. The pair has gradually become more and more difficult, the separation decreasing from 0″·6 to 0″·2. There is at the present time a rapid orbital motion, but it will be long before an accurate period can be deduced, as the early observations do not cover the time of maximum separation. All the observations are reasonably satisfied except Schiaparelli's of 1888, which leaves a residual of 14°·5. Burnham's observations of the following year leave a residual of 1°·9 with the opposite sign. This is a difficult pair, well deserving observation at the present time.

T	1919·37	a	0″·407	i	37°·0
P	216·9 years	e	0·545	Ω	106·5
n	−1°·660			ω	350·0

1915	121°·8	0″·188
1920	95·0	0·182
1925	66·3	0·177
1930	38·1	0·186
1935	14·2	0·210

The hypothetical parallax is ″·0089.

2093. OΣ77. B.D. 31°, 737.

R.A. $4^h\ 11^m\ 9^s$. Mags. 7·5, 7·5. P.M. ″·051 in 188°·7 (Greenwich).
Dec. +31° 31′. Spec. F8.

As the components of this double star are of about equal brightness, and as the pair is close, the observations are all uncertain as to quadrant. Previous to Burnham's observations in 1878 there were observations on five nights only. The observation by Secchi in 1859 cannot be reconciled with the others. The mean of the four observations by Otto Struve and Maedler was taken as defining one position-angle. The result cannot, however, be regarded as satisfactory, for Otto Struve's and Maedler's observations show if anything a retrograde motion in six years, while the orbit given below shows an increase of 37° in the same period. The representation of the early observations is as follows:—

	Obs.	Comp.	O−C.	Obs.	Comp.	O−C.	
	°	°	°	″	″	″	
1846·06	338·2	326·7	+11·5	0·37	0·23	+·14	OΣ 2
46·12	330·1	327·1	+ 3·0	0·35	0·23	+·12	Ma 1
52·19	337·3	4·8	−27·5	0·3±	0·28	+·02	Ma 1
59·05	250±	29·8	..	...	0·33	...	Se 1
78·50	69·3	69·2	+ 0·1	0·44	0·43	+·01	β 4

The observations after 1878 are represented as well as possible, but the elements are all rather uncertain. The orbit is nearly circular. It was deduced by Zwiers' method.

T	1917·21	a	″·44	i	58°·2
P	95·0 years	e	0·050	Ω	55°·3
n	+3°·788			ω	174°·8

1915	227°·6	″·409
1920	238·8	0·416
1925	250·5	0·388
1930	265·4	0·331
1935	287·3	0·268

The hypothetical parallax is ″·0168.

2383. β552. B.D. 13°, 728.

R.A. $4^h\ 47^m\ 35^s$. Mags. 7·0, 10·0. P.M. No evidence of P.M. (Porter).
Dec. +13° 32′. Spec. F5.

There is a previous orbit of this binary by See (*M.N.*, **68**, 198). The recent measures show that the distance is increasing and the angular motion decreasing as compared with the ephemeris based on See's orbit. His period of 56 years is therefore too small.

The angular motion in the 38 years to 1912 is about 270°, but the elements obtained below show that a longer period will be required to complete the remaining 90°. There were no reliable observations between 1877 and 1889, during which interval 145° of the 270° were described. The early observations of Burnham in 1874 and 1877 seem to be fundamentally correct, and have been satisfied as follows:—

	Obs.	Comp.	O−C.	Obs.	Comp.	O−C.
1874·95	340°±	337°·3	+2°·7	0″·5±	0″·37	+″·13
1877·97	360°±	6°·3	−6°·3	0″·8±	0″·25	+″·55

The first of these observations was made with the 6-inch telescope, and the note added "possibly a close pair." The second of these observations, which is considered as the real discovery observation, was made with the 18½-inch telescope of the Dearborn Observatory. The orbit has been calculated from the following positions:—

1875	340°·0	0″·360
...	60°·0	0″·214
1889	144°·0	0″·370
1901	208°·7	0″·443
1913	248°·0	0″·625

The elements given below do not satisfy the observation of H. Struve in 1886, and the single observation by Burnham in 1903 leaves a residual of 13°·1. All other observations except one are satisfied within 7°. On account of the difference in magnitude of the components, this is at all times a difficult pair, so that on the whole the observations may be considered well represented.

T	1882·85	*a* 0″·627	*i* 50°·4
P	88·2 years	*e* 0·519	Ω 122°·0
n	+4°·083		ω 316°·3

1915	251°·2	0″·658
1920	260°·4	0″·728
1925	268°·0	0″·787
1930	274°·4	0″·827
1935	280°·8	0″·851

The hypothetical parallax is ″·0251.

5223. OΣ208. B.D. 54°, 1331.

R.A. $9^h\ 47^m\ 1^s$. Mags. 5·0, 5·6. P.M. ″·009 in 360° (Boss).
Dec. +54° 25′. Spec. A2.

In 1875 Casey determined the period as 115·4 years. Later computers reduced the period considerably, Glasenapp finding

91·92 years, See 97·0, and Doberck 97·70. The period of 110·1 years is probably correct within two or three years.

The apparent orbit is based on the following positions :—

1844	7°·1	0″·385
1862	46·3	0·288
...	135·0	0·176
1894	259·0	0·218
1912	308·6	0·388

and the resulting elements are—

T	1883·61	a	0″·319	i	32°·2
P	110·1 years	e	0·473	Ω	124·6
n	+3°·268			ω	44·5

1915	312°·9	0″·403
1920	320·2	0·424
1925	326·8	0·436
1930	333·2	0·442
1935	339·6	0·440

The hypothetical parallax is ″·0110.

6158. Σ1639. B.D. 26°, 2345.

R.A. $12^h\ 20^m\ 40^s$. Mags. 6·7, 7·9. P.M. ″·022 in 120°·3 (Greenwich).
Dec. +26° 0′. Spec. A5.

This double star has previously been investigated by Lewis (*M.N.*, 62, 209), who obtained a period of 180 years. The observations of the present time show that the period must be much longer. When first observed by Struve in 1828 the separation was over 1″. The angle decreased slowly and the distance decreased rapidly till in the eighties it became too close for observation. It was again observed in 1895, when the angle had changed about 220° from the position in which the companion was last seen. The distance has been steadily increasing since. About 310° have been described in 90 years, but according to the orbit given below it will require 270 years to complete the remaining 50°. The period must therefore be largely uncertain. The eccentricity obtained is very large, but cannot be much in error. If the period has to be increased so also will the semi-major axis, but it is probable that the value of the hypothetical parallax deduced from the elements given below will be nearly correct.

For orbits of this type, in which the motion is at times mostly in distance and at other times mostly in angle, Thiele's method

is very convenient for computing the orbit. For the determination of the orbit the three following complete observations were used :—

1830	292°·4	1″·195
1880	265·8	0·380
1918	340·7	0·670

Practically no attention was paid to the observations of 1887, 1888, 1895. The computed separation for these dates is 0″·06, 0″·06, 0″·26. Otherwise the observations are well satisfied.

T	1888·1	a	1″·00	i	43°·6
P	361 years	e	0·9258	Ω	78·4
n	−0°·997			ω	59·1

1915	342°·8	0″·595
1920	339·3	0·666
1925	336·5	0·730
1930	334·2	0·786
1935	332·2	0 838

The hypothetical parallax is ″·0157.

6566. Σ1768. B.D. 37°, 2433.

R.A. $13^h\ 34^m\ 8^s$. Mags. 5·7, 7·6. P.M. ″·104 in 277°·7 (Boss).
Dec. +36° 40′. Spec Fo.

Doberck in 1880 obtained for this binary a period of 119 years, but in a new determination in 1910 he increased the period to 553 years. The orbits by See and Lewis have periods of 184 and 220 years. From 1830 to 1858 the separation decreased from 1″·2 to 0″·3. Between 1859 and 1872 the star appeared single in the Pulkowa refractor. The computed distance for 1876, when the duplicity was first reobserved, is 0″·6, and the distance is now greater than it was in 1830. The eccentricity is large, and as given below must be nearly correct. Although the period is still very uncertain, the other elements are probably fair approximations, except that the semi-major axis will have to be increased with the period. The apparent orbit has been deduced from the following places :—

1832	76°·4	1·190
1856	38·0	0·347
...	283·1	0·133
1876	162·1	0·618
1904	129·6	1·141

The resulting elements are—

T	1860·26		a	1″·205	i 47°·4
P	220·4 years		e	0·8562	Ω 52·8
n	−1°·6335				ω 241·4

1910	126°·4	1″·218
1920	121·4	1·343
1930	117·1	1·450
1940	113·4	1·539

The hypothetical parallax is ·0262.

6641. Σ1785. B.D. 27°, 2296.

R.A. $13^h\ 45^m\ 53^s$. Mags. 7·2, 7·5. P.M. ″·491 in 260°·9 (Greenwich).
Dec. +27° 21′. Spec. K2.

When Struve first observed this pair the separation was 3″·5 and the angular motion only about 0°·7 per annum. The distance gradually decreased, and for the last 20 years has been nearly constant at 1″·36, with an annual change in position-angle of more than 4°. It is always an easy pair to observe, and there are many observations now extending over an arc of 200°. The orbit may therefore be considered as fairly well determined. The elements given below satisfy the observations with a mean discordance of well under 1° in position-angle. There is a recent orbit by v. Biesbroeck (*Ann. Roy. Obs. Belgique*, 9, 112) with a period of 199·2 years. The new period must be correct within a very few years.

T	1913·29		a	2″·55	i 39°·4
P	193·55 years		e	0·460	Ω 156·3
n	+1°·860				ω 180·5

1915	342°·7	1″·374
1920	4·9	1·356
1925	27·6	1·357
1930	49·2	1·422
1935	68·0	1·562

The hypothetical parallax is ″·0605.

6999. Σ1879. B.D. 10°, 2739.

R.A. $14^h\ 42^m\ 36^s$. Mags. 7·8, 8·8. P.M. ″·272 in 165°·2 (Porter).
Dec. +9° 58′. Spec. F8.

The separation of the components was over 1″ in Struve's measures. The distance rapidly decreased, and after the observa-

tion by Maedler in 1851 the star appeared single for 25 years, the measures of 1865 and 1867 being unsupported by later observations. In 1877 the star was observed as double by Burnham, Schiaparelli, and Dembowski, the separation being then just under 0″·4. Since then there have been fairly numerous observations.

In *Ann. Roy. Obs. Belgique*, 9, 119, v. Biesbroeck gives an orbit with a period of 238 years. In the orbit given below the period is reduced to 177·9 years. The following tables give the computed places for a few dates corresponding to the two orbits :—

	v. Biesbroeck's Orbit.		New Orbit.	
1829·99	67°·7	1″·14	67°·3	0″·96
1851·41	47·5	0·57	42·3	0·50
1877·78	218·1	0·36	214·8	0·35
1889·78	164·8	0·43	164·8	0·44
1902·90	133·6	0·65	133·6	0·60
1920·00	115·6	0·95	113·7	0·85

The observations in angle are not sufficiently accurate to discriminate between the two sets of elements. With regard to distances, the new orbit does not fit Struve's measures so well, but fits recent measures better.

T	1868·14	a	0″·789	i	51°·2
P	177·9 years	e	0·623	Ω	70·6
n	−2°·023			ω	211·7

1915	118°·3	0″·784
1920	113·7	0·849
1925	109·6	0·910
1930	106·1	0·967
1935	103·0	1·018

The hypothetical parallax is ″·0198.

7001. OΣ285. B.D. 42°, 2531.

R.A. $14^h\ 42^m\ 40^s$. Mags. 7·1, 7·6. P.M. ″·048 in 170°·5 (Greenwich).
Dec. +42° 41′. Spec. F5. $\pi - 0''{\cdot}028 \pm {\cdot}015$ (Sproul).

This double star has described the greater part of a revolution since 1843, when it was first measured. The observations of Maedler and Otto Struve cover the period 1843 to 1855, when the separation was just under half a second. The corrections to his observations found by Otto Struve have been applied, and this makes the early observations in fair agreement. The observations by Dembowski in 1865 and Burnham in 1876 are also well represented. In the past considerable importance has generally

been given to the observations made by Engelmann on five nights in 1883. However, for that date the computed separation was only 0″·15 (measured 0″·22), and Engelmann had only a seven-inch telescope. The position-angles have a range of 40°, and no attempt was made to satisfy them. The mean is actually represented to 16°, which is the motion in one year at that epoch. All the observations after 1883 are closely represented.

The orbit by v. Biesbroeck has a period of 97·93 years. This is considerably reduced in the new orbit. The arc covered by the observations 1843 to 1917 is nearly 340°, so that the elements must be pretty accurate. The uncertainty is mostly due to personality in the observations, for at the time of the early observations the motion was only 1°·4 a year, while the correction applied to Otto Struve's observations is about 9°. The orbit was deduced by Zwiers' method.

T	1882·64	a	0″·33	i	25°·6
P	88·5 years	e	0·553	Ω	41·7
n	−4°·067			ω	222·3

1915	98°·7	0″·450
1920	90·9	0·473
1925	83·7	0·488
1930	76·8	0·493
1935	69·9	0·488

The hypothetical parallax is ″·0132.

7259. Σ1938. B.D. 37°, 2636.

R.A. $15^h\ 21^m\ 42^s$. Mags. 6·7, 7·3. P.M. ″·172 in 302°·7 (Boss).
Dec. +37° 36′. Spec. Ko. π +″·054 ± ·011 (Sproul).

Many orbits for this double star have been computed. The most recent is by Lohse, who obtained a period of 244 years. Since Struve's observations as much as 280° of the apparent orbit has been described, but this has been more or less symmetrically disposed about periastron, and represents only about one-third of the period. The elements given below have been obtained with the help of the Herschel observations of 1782 and 1802, when the separation was not far short of the maximum. The apparent orbit was determined from the following five positions:—

1782·6	356°·8	1″·850
1835·0	318·1	1·000
1855·0	248·6	0·510
1885·0	111·8	0·697
1915·0	52·4	1·197

The resulting elements represent all the observations very well. The accuracy of the period depends largely on the accuracy of Herschel's observations. If Herschel's observations are correct the period will be correct within 10 years or less, the eccentricity will be nearly correct, and the angular elements will be within a few degrees of the truth.

T	1865·04	a	1″·271	i	40°·9
P	234·8 years	e	0·560	Ω	174°·5
n	−1°·533			ω	22°·1

1915	52°·6	1″·194
1920	47°·5	1·278
1925	43°·1	1·362
1930	39°·2	1·440
1935	35°·6	1·512

The hypothetical parallax is ″·0265.

7649. Σ2055. B.D. 2°, 3118.

R A. 16h 27m 8s. Mags. 4·0, 6·1. P. M. ″·097 in 209°·7 (Boss).

Dec. +2° 8′. Spec. Ao. π { +″·018 ± ·003 (Yale). −″·010 ± ·008 (Allegheny).

Between 1825 and 1919 the change in position-angle has amounted to 110°, about equally distributed about the position of maximum separation. Although the period is only 110 years, no good orbit has yet been obtained from observations beginning with Struve. There are two recorded observations made by Herschel in 1783 and 1802. With the aid of these observations computers, beginning with Struve himself, have worked on this system. The position-angles given by Herschel on the above dates are 75°·5 and 69°·3. The motion, however, is direct, and on account of the great difference in the magnitudes of the components there is no reason to assume a correction of 180°. Herschel recorded his observations for the star as 14°·5 *nf* and 20°·7 *nf*, and it has generally been assumed that the letters describing the quadrant in one or both observations have been erroneously recorded. Glasenapp (*M.N.*, 48, 254) actually discussed all the 16 possible combinations of the letters and deduced a period of 373 years. Lewis (*Memoirs R.A.S.*, 56, 454), adhering to the first measurement and changing the second to 110°·7, has obtained a period of 134 years. Lohse (*Potsdam Publ.*, 20, 128), discarding Herschel's observations altogether, reduced the period to 123 years. The difficulty is

solved in Herschel's *Collected Works*, **1**, 180, footnote, where we read, relative to the observation of 1783: "The measure was forgot to be wrote down but it being the last taken I find my micrometer stands at 14° 30′ which agrees well enough with the figure, but as the instrument has been touched a good deal it may have been altered." In the orbit given below the 1802 observation is satisfied to 0°·9. The position-angle in 1783 is 48°·0 and the distance 1″·44, showing that Herschel recorded the quadrant correctly and that the separation of the components was such as to make the discovery easy. An approximate orbit computed with modern observations only, shows that it would have been difficult for Herschel to have discovered the companion in any but the first quadrant.

The values of P, T, Ω, ω given below must be nearly correct, but the elements a, e, i are closely connected with the nearness of approach of the two stars. In the next 20 years the companion should describe the 240° of the apparent orbit still unobserved and return to the Struve position of 1830. Observations of this binary should be carefully made in that interval.

T	1927·4	a	1″·328	i	53°·2
P	110·3 years	e	0·86	Ω	110°·0
n	+3°·264			ω	96°·7

1915	72°·0	1″·023
1920	82·4	0·809
1925	105·5	0·437
1930	304·2	0·433
1935	330·5	0·728

The hypothetical parallax is ″·0459.

7748. Δ15. B.D. 43°, 2639.

R.A. 16h 41m 37s. Mags. 8·0, 8·2.
Dec. +43° 37′. Spec. K5.

This orbit has previously been investigated by Gore, Evans, Lewis, and See, who obtained periods ranging from 70 to 205 years. The observations now extend over 240° of position-angle. The maximum separation of the components has not been observed. The observations extend over rather less than 50 years, and as the period obtained below is 126 years there must be considerable uncertainty. The elements have been deduced by Glasenapp's

adaptation of Kowalsky's method (*cf.* See, *Evolution of Stellar Systems*, 1, 49).

T	1894·52	a	0″·935	i	59°·3
P	126·1 years	e	0·435	Ω	147°·1
n	−2°·855			ω	213°·0

1915·0	262°·2	0″·486
1917·5	250°·0	0·489
1920·0	238°·3	0·506
1922·5	227°·6	0·533
1925·0	218°·2	0·573
1927·5	210°·1	0·619
1930·0	203°·2	0·671

The hypothetical parallax is ″·0295.

7783. Σ2107. B.D. 28°, 2624.

R.A. $16^h\ 48^m\ 52^s$. Mags. 6·5, 8·0. P.M. ″·029 in 33°·3 (Greenwich).

Dec. +28° 47′. Spec. F5. π { +″·006 ± ·006 (Sproul). + ·021 ± ·008 (Yerkes).

The orbit of this double has been investigated by Casey, Berberich, Lewis, Lohse, and Rabe. The uncertainties in the orbit should soon be removed, as the observations extend over 240° in position-angle and over half of the period. The early observations seem to cover the period of maximum separation. The apparent orbit has been based on the five following positions:—

1830	149°·8	1″·02
1860	182·9	0·91
1880	221·4	0·61
1895	289·2	0·42
1910	9·8	0·54

The elements are—

T	1900·04	a	0″·783	i	34°·3
P	154·5 years	e	0·387	Ω	0°·6
n	+2°·33			ω	316°·7

1915	24°·8	0″·572
1920	40·5	0·604
1925	54·6	0·635
1930	67·5	0·669
1935	79·0	0·705

The hypothetical parallax is ″·0216.

9319. Σ2525. B.D. 27°, 3391.

R.A. $19^h\ 23^m\ 29^s$. Mags. 7·4, 7·6. P.M. ″·127 in 46°·6 (Greenwich).
Dec. +27° 10′. Spec. F8.

Although the position-angle has changed by 310° since the observations of Struve, the period is still very uncertain. Orbits by Gore, Bowyer, and Doberck give periods of 138·5, 306·7, and 243·9 years respectively. The orbit given below has a period of 354·9 years. For many years to come the predicted position-angles of the new orbit will sensibly agree with those of Doberck's orbit, but the distances will differ considerably. Improvement of the elements will largely depend on accurate measurement of the separation. The separation was 1″·3 at discovery: it steadily decreased till 1887, when the star appeared single in all telescopes; it is now approaching 1″, and will for long continue to increase. The eccentricity is very large: it is quite definitely greater than 0·9.

The shape and position (but not the size) of the apparent orbit has been deduced from the following normal places:—

1830	256°·3	1″·33
1860	244·4	0·88
1880	220·0	0·40
1892	340·5	0·29
1917	303·7	0·98

The final elements are—

T	1887·31	a	1″·125	i	37°·5
P	354·9 years	e	0·9332	Ω	1·0
n	−1°·014			ω	93·4

1915	304°·9	0″·808
1920	302·3	0·886
1925	300·2	0·959
1930	298·3	1·026
1935	296·7	1·088

The hypothetical parallax is ″·0178.

9605. Σ2579. B.D. 44°, 3234.

R.A. $19^h\ 42^m\ 38^s$. Mags. 3·0, 7·9. P.M. ″·c66 in 55°·1 (Boss).
Dec. +44° 56′. Spec. A. π +″·049 ±″·008 (Sproul).

The arc described is not yet sufficient for an accurate determination of the orbit. Only about 110° have been described by the companion since Struve's time. Herschel made a single observation in 1783, but he says it is not very good. Yet as

this observation extends the arc by 32° and the time interval by nearly 50 years, it must be considered in the determination of an orbit. The elements given below can only be considered to be approximate, although they fit the observations very well.

The orbit given below is based on the following positions (referred to the equinox of 1900) :—

1783·72	70°·8	2″·51
1830·0	38·7	1·92
1860·0	359·7	1·54
1885·0	321·7	1·56
1916·0	282·1	1·79

The elements are—

T 1941·6	*a* 2″·12	*i*	47°·8
P 321·0 years	*e* 0·188	Ω	87·9
n −1°·122		*ω*	159·0

1915	283°·2	1″·726
1920	277·7	1·742
1925	272·3	1·746
1930	266·8	1·737
1935	261·2	1·712

The hypothetical parallax is ″·0359.

9650. OΣ387. B.D. 34°, 3727.

R.A. $19^h\ 45^m\ 55^s$. Mags. 7·2, 8·2. P.M. ″·119 in 44°·9 (Porter).
Dec. +35° 7′. Spec. F5.

This was rather a difficult pair for the early observers, and the observations do not appear to be very accurate. The total arc described since 1844 is about 170°. The period is probably of the order of 130 years. The minimum separation of o″·4 was reached about the time of Dembowski's earlier observations, and the maximum separation of o″·6 seems to have occurred about 1903. The pair will therefore always be a fairly difficult one for any but the largest telescopes. The orbit has been based on the following data :—

1843	123°·0	0″·49
1861	72·5	0·40
1879	16·4	0·48
1897	343·5	0·61
1915	316·6	0·57

The elements are—

T	1946·7	a	0″·566	i	51°·5
P	128·0 years	e	0·179	Ω	146°·4
n	−2°·813			ω	55°·0

1915	316°·4	0″·569
1920	307°·1	0·515
1925	295°·1	0·446
1930	278°·4	0·361
1935	254°·3	0·313

The hypothetical parallax is ″·0177.

10559. Σ2729. B.D. − 6°, 5604.

R.A. 20h 47m 27s. Mags. 5·9, 7·2. P.M. ″·090 in 97°·0 (Boss).
Dec. − 5° 54′. Spec. F.

There are recent orbits of this pair by Aitken (*Publ. A.S.P.*, **21**, 84, and *Publ. L.O.*, **12**, 153) and Doberck (*A.N.*, **190**, 199) giving periods of 135·6 and 646·5 years respectively. Both orbits fit the modern observations within their probable errors. The difference arises in the way the early observations are treated. The discordances in the observed angles are enormous, amounting to 60°. Aitken rejected all the observations of Maedler, the observations by Herschel in 1802, and the observations by Struve in 1830, 1832, 1833. His orbit is based on the observations by Sir John Herschel in 1825, 1832, and the considerable series of observations by Dawes from 1839 to 1854. The early observation of Herschel in 1783 is shown with a residual of 13°·2. Doberck, on the other hand, attempted to use all the observations, and the result shows probable errors of 8°·6 in Struve, 19°·5 in Dawes, and 12°·2 in Maedler, although the residuals are in many cases much larger and systematic for the two latter observers. The 1783 observation of Herschel is represented to 15°·4 in the opposite direction from Aitken's orbit. The separation in 1783 was 0″·56 according to Aitken and 1″·47 according to Doberck.

The orbit given below was deduced by Zwiers' method, and satisfies the early observations as follows :—

	Obs.	Comp.	O − C.	Obs.	Comp.	O − C.	
	°	°	°	″	″	″	
1783·55	351·5	351·9	− 0·4	...	0·78	...	H 1
1802·66	28·9	4·5	...	...	0·76	...	H 1
1825·60	27·5	26·9	+ 0·6	0·80	0·53	+·27	Σ 2
1832·73	46·0	39·5	+ 6·5	0·67	0·45	+·22	h 2, 1
1833·34	27·1	40·8	−13·7	0·67	0·43	+·24	Σ 2, 1
1834·69	45·0	43·9	+ 1·1	0·5	0·42	+·08	Sm 1
1836·05	46·3	46·9	− 0·6	0·41	0·41	·00	Σ 4
1840·20	63·8	58·4	+ 5·4	0·6±	0·37	+·23	Da 4, 2
1841·80	72·7	63·2	+ 9·5	...	0·36	...	Da 2
1843·76	81·7	69·5	+12·2	...	0·35	...	Da 1
1854·22	98·8	106·6	− 7·8	0·4±	0·36	+·04	Da 2

The observed angles of Maedler are all 30° to 50° smaller than the computed values. It will be apparent that the orbit is still very uncertain. The ephemeris below is referred to the equinox of date.

T	1897·22	*a*	0″·70	*i*	67°·4
P	151·7 years	*e*	0·375	Ω	167°·8
n	+2°·374			ω	59°·7

1915	331°·0	0″·471
1920	338°·4	0·588
1925	343°·6	0·678
1930	347°·6	0·741
1935	351°·2	0·782

The hypothetical parallax is ″·0195.

12290. β80. B.D. 4°, 4994.

R.A. $23^h\ 15^m\ 2^s$. Mags. 8·2, 9·1. P.M. ″·492 in 102°·9 (Porter).
Dec. +5° 0′. Spec. G8. π + ″·005 ± ″·007 (McCormick).

The latest measures of this pair show an angular motion of 300° since the time of discovery. However, the apastron is not well defined by the early measures. If the early angles were correct the maximum separation would have taken place about 1885, but the measured distances show that the maximum separation took place earlier, probably about the time of discovery in 1875. The following elements satisfy the observations pretty well from 1875 to 1919. The orbit was deduced by Zwiers' method.

T	1904·69	*a*	0″·79	*i*	43°·0
P	85·7 years	*e*	0·773	Ω	174°·1
n	+4°·201			ω	288°·9

1915·0	235°·1	0″·592
1917·5	242°·9	0·657
1920·0	249°·3	0·717
1922·5	254°·8	0·772
1925·0	259°·6	0·819
1927·5	263°·9	0·855
1930·0	267°·8	0·900

The hypothetical parallax is ″·0323.

Erratum.

In Dr. Fotheringham's paper, page 305, line 23 :—*for* 25″·22 *read* 25″·52.